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A STOCHASTIC MODEL FOR THE CRYSTALLIZATION AND TEXTURAL ANALYSIS OF A FINE GRAINED GRANITIC STOCK NEAR MEECH LAKE, GATINEAU PARK, QUEBEC

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CONTENTS

1	Abstract/Résumé
1	Introduction
2	Geology and petrology
11	Stochastic model of aplite melt crystallization
13	Statistical testing of the model
13	Hypothesis of homogeneity
13	Markov hypotheses
14	The parameters c_{ij}
15	Analysis of observations
15	Materials under investigation
21	Meech Lake aplite and ideal granite model
22	Nonideal granites with Markov chains of the first order
22	Alteration of aplitic rocks and carbonatites
24	Geological interpretation of the parameters c_{ij}.
26	General conclusions
28	Acknowledgments
28	References

Appendixes

31	1.	FORTRAN IV computer program MARKOV
52	2.	Method of counting transitions
54	3.	Matrices of transition frequencies for sequences of grains
61	4.	Individual values of c_{ij} of the elements of matrix C

Tables

6	1.	Modal composition of potassic aplite near Meech Lake
15	2.	Data block and some results for sample No. 25
16	3.	Sequences of grains of orthoclase, quartz and plagioclase similar to first-order Markov chains (ideal granites)
18	4.	Sequences of grains of orthoclase, quartz and plagioclase similar to first-order Markov chains (non-ideal granites)
19	5.	Sequences of grains of orthoclase, quartz and plagioclase similar to second-order Markov chains
24	6.	Contingency table for type of sequence of grains and form of aplite body
25	7.	δ_C values for Gatineau aplites similar to ideal granites
26	8.	δ_C values for other granitic rocks

Figures

3	1.	Geological map of Meech Lake aplite and surrounding area
4	2a,b.	Contacts of aplite with host rock (syenite gneiss)
5	3.	Triangular diagram showing relative proportions of microcline, plagioclase, and quartz of 145 aplite samples
8	4.	Modal percentage plagioclase in Meech Lake aplite
9	5.	Sum of modal percentage values for pyroxene and amphibole in aplite
10	6.	Part of aggregate of amphiboles in aplite
10	7.	Typical Meech Lake aplite texture
13	8.	Typical subsequence of grains in Meech Lake aplite
20	9.	Confidence intervals for probabilities P_i and P_{ii} for microcline, quartz and albite in aplite stock
21	10.	Confidence intervals for probabilities P_i and P_{ii} for microcline, quartz and albite in aplite dykes
23	11.	Nonideal granite samples in Meech Lake aplite stock
27	12.	Contour map of the viscosity index δ_C

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Abstract

A stock of fine grained potassic granite (Meech Lake aplite) has been investigated using a stochastic model for 'ideal' granite developed at the Laboratory of Mathematical Geology in Leningrad. From observations of the contacts of the stock with its country rocks, it can be assumed that the stock originated from a melt and that crystallization took place in a magma chamber. A hypothesis on the absence of significant metasomatic alteration in the stock was tested and confirmed through the investigation of about 140 specimens collected from the stock. This conclusion does not contradict traditional petrographic analysis.

Because the stochastic-process model of a magmatic, relatively unaltered, ideal granite fits the observations well, the parameters of this model were investigated in detail. These parameters can be indicative of the viscosity of the magma. The pattern of parameter values for the stock shows that different parts of the stock were generated from the same magma but with differences in viscosity and in content of volatile components.

Résumé

Un massif intrusif de granite potassique à grains fins (aplite de Meech Lake) a été étudié au moyen d'un modèle stochastique propre à un granite "idéal" mis au point au Laboratoire de géologie mathématique, à Leningrad. A partir des observations faites des contacts du massif intrusif avec ses roches encaissantes, on peut supposer qu'il doit son origine à une fusion, et que la cristallisation s'est produite dans la chambre magmatique. Une hypothèse avan ant l'absence d'altération métasomatique importante du massif intrusif a été vérifiée et confirmée grace à l'étude de 140 spécimens prélevés sur le massif. Cette conclusion ne contredit pas l'analyse pétrographique traditionnelle.

En raison du fait que le modèle à processus stochastique d'un granite idéal, de nature magmatique et peu altérée, s'accorde bien avec les observations recueillies, les paramètres de ce modèle ont été étudiés en détail. Ils peuvent donner une indication de la viscosité du magma. Le modèle des valeurs des paramètres particuliers au massif montre que différentes parties du massif intrusif proviennent du meme magma, mais que ce dernier accuse des différences dans le degré de viscosité et le contenu de composantes volatiles.

INTRODUCTION

The theory of stochastic models for the crystallization and metasomatic alteration of magmatic granitic rocks has been developed at the Laboratory of Mathematical Geology in Leningrad during the past 12 years (Vistelius, 1972, 1980). Investigations on granitic rocks in different regions have been made for checking the consistency of the stochastic models with data observed in the field.

During this time two groups of granitic rocks have been studied. The first group comprises intrusions that do not contain any ore deposits, such as the Marlsburg massif, Schwartz Wald (Federal Republic of Germany) and the Kyzyl-tas massif in Kazakhstan (Ivanov, 1978). The second group consists of granites containing ore deposits within the granitic body itself or at its contacts; this group includes the Carmenellis granite in Cornwall, England, the Omsukchan massif near the Kolyma river in northeastern Asia (Vistelius and Romanova, 1972), the Iul'tin intrusion located in Chukotka (Ivanov and Podol'sky, 1974), and the Arga-Ynykh-Khaya crescent dyke in Yakutya (Romanova, 1978).

These studies have shown that a relatively simple stochastic model of crystallization called the ideal granite model applies to granites which were generated by magmatic means and which were not subjected to metasomatic alteration involving the main rock-forming minerals (quartz, potassium feldspar, and plagioclase).

If granitic rocks were metasomatically altered, as can be observed for most granites which contain high-temperature hydrothermal deposits such as quartz-cassiterite veins, and if secondary quartz, potassium feldspar or plagioclase were generated, these granites have textures which are quite different from those of the barren unaltered magmatic granites and other, more complex, stochastic models have to be applied.

More recently it has become clear that it is important to determine if in fact granitic massifs exist which are essentially composed of rocks with textures as predicted by the simple stochastic model of 'ideal' granite. These would be unaltered ideal granites. Another problem of interest is the investigation of the possible influence of younger carbonatite veins on unaltered granitic rocks intruded by these veins. The solution of this problem is important in petrology and economic geology since it should be possible to use the result to draw conclusions about the interrelationships between hydrothermal processes and the processes by which the carbonatites have been generated.

These problems were discussed by the authors in June 1975 in Ottawa, and a fine grained granitic stock (Meech Lake aplite) in Gatineau Park, Quebec, was selected for investigation because some thin sections from it showed the characteristics of 'ideal' granite and because it contains younger carbonatite dykes. Initially, about 60 new aplite samples were collected and investigated in the Laboratory of

Mathematical Geology in Leningrad. This preliminary investigation indicated that the rocks of the Meech Lake stock could indeed be used for a study contributing towards the solution of the preceding problems. More sampling was required and eventually about 140 samples were collected and studied. Additionally, detailed observations on the contact of the stock and its country rocks were performed as well as a study of the rock-forming minerals. In 1977, F.P. Agterberg visited the Laboratory of Mathematical Geology in Leningrad and the later stages of this joint project were discussed. The present paper contains the final result of our investigations.

GEOLOGY AND PETROLOGY

The Meech Lake stock lies in the southeastern part of Gatineau Park, Quebec, about 16 km northwest of Hull (Fig. 1). This aplite body can be reached, on foot, by taking McCloskey's Road south 1.1 km from Meech Lake or by walking the Gatineau Parkway extension, 1.0 km beyond the end of pavement at Champlain Lookout (Fig. 1).

Although the area has been the subject of several early geological reconnaissance surveys, the first references to aplite seem to have been made by Béland (1951). He referred to the aplite-pegmatite suite as the Meech Lake Complex, a name proposed informally by F.F. Osborne. The aplite bodies were delineated and briefly described by Hogarth (1966). Further description is given by Béland (1977, p. 43-46). The Meech Lake area falls within the limits of the map sheets of Hogarth (1970) and Béland (1977).

The aplite bodies are within the southern part of the Precambrian Wakefield Batholith which cuts metasedimentary rocks. This batholith outcrops over an area of 285 km^2 and extends from the Bristol-Masham area of Sabourin (1965) eastward into the Wakefield area of Béland (1977).

The Wakefield Batholith includes rocks which range in composition from granite to gabbro, but syenite and monzonite are the most common types near Meech Lake. The rocks have been notably affected by metamorphism, with a weak to strong foliation discernible in most outcrops. South of Meech Lake, foliations are roughly parallel to axial surfaces which trend northeasterly and dip to the southeast. The rock is referred to as orthogneiss in this paper.

Unzoned granite to syenite pegmatites intrude orthogneiss throughout the region. The larger dykes, which are up to 2.75 km long, trend mainly northwest and northeast and dip steeply, regardless of local changes in foliation of the hostrocks. Near Meech Lake the pegmatites, especially the smaller dykes, tend to be syenitic.

Orthogneiss and pegmatite, in turn, have been intruded by aplite although this sequence is complex. The aplite, itself, is cut by numerous soda-amphibole veins, carbonatites and explosive breccias (Béland, 1951; Sabourin, 1952; Hogarth, 1966). The K-Ar age of the aplite can be bracketed by mineral ages on orthogneiss and carbonatites. A hornblende age of orthogneiss from Artist's Point (3.7 km southwest of the centre of Fig. 1) gave 978 Ma (Wanless et al., 1972); phlogopite ages of two carbonatites cutting the main aplite stock of Figure 1 gave 910 and 930 Ma (Hogarth, 1966).

The relationship between aplite and pegmatite is confusing. In most cases aplite seems later than pegmatite, but some pegmatite dykes (such as those shown in Fig. 2) cut the aplite. A few aplites are transitional to pegmatites with groups of giant feldspar and large quartz grains in indistinct contact with aplite. The gross mineralogy of these various age groups of pegmatites seems similar to that of aplite, in that they are essentially composed of perthitic microcline, oligoclase and quartz; but the relative amounts of those minerals vary widely. Some peculiarities, however, occur in the minor minerals: yttrian pegmatites, characterized by yttrian garnet (Kasowski and Hogarth, 1968), are pre-aplite; cerian pegmatites characterized by allanite and uranothorite, are post-aplite.

The carbonatites are dolomitic to calcitic, arcuate-tabular bodies. Commonly dolomitic carbonatites are separated from aplites by a zone of fine grained phlogopite replacing microcline. The carbonate becomes calcitic near the border suggesting the reaction:

$$3\,Ca\,Mg\,(CO_3)_2 + KAl\,Si_3O_8 + H_2O \rightleftharpoons$$

Dolomite Orthoclase

$$3\,CaCO_3 + KMg\,AlSi_3O_{10}(OH)_2 + 3CO_2$$

Calcite Phlogopite

as proposed for the Rufunsa region of Zambia by Bailey (1966). Layers of white sugary apatite are uncommon. Near the borders of the carbonatite, isolated crystals of sodic amphibole are especially abundant in the phlogopite selvage. The maximum temperature of equilibration of calcite with intergrown dolomite crystals was determined at 610°C by Wong (1971, p. 42) using the calcite solvus method.

Aplite constitutes a rather large proportion of the outcrop south of Meech Lake (Fig. 1). By far the largest body is shaped like a jelly-fish with the body pointing westwards and tentacles (as aplite apophyses) straggling towards the east. Across its width, the stock measures 1.7 km maximum. Most of the smaller bodies lie approximately parallel to the foliation of the surrounding orthogneiss, but even the large body appears to be extended northeast-southwest, approximately parallel the regional trend.

Contacts with the host rock, where observed, are sharp. In some cases, however, there is an alternation of small bodies of aplite and orthogneiss near the contact of a major aplite mass. Commonly the aplite-orthogneiss contact is marked by a selvage of phlogopite-amphibole. This is well shown at the falls on McCloskey's Creek (Fig. 2a) where the aplite border clearly transects the foliation of the gneiss with a discordance of about 25°. Another contact, about 100 m southwest, is shown in Figure 2b. In other places a carbonatite dyke separates aplite from orthogneiss. More commonly the contact is obscured by overburden and a 'defined geological boundary' on Figure 1 indicates a contact that can be located within a width of 15 m.

Xenoliths of country rock are rare within the aplite. A possible large xenolith of orthogneiss is marked in the northwestern part of Figure 1, but contacts are poorly defined. Blocks of orthogneiss were observed in some explosive breccias within the aplite.

Figure 1 shows the main body of aplite (aplite stock) surrounded by smaller aplitic bodies. These bodies are elongated and in some cases (such as the body containing sample 142) they are typical dykes which cut the country rocks.

Table 1 and Figure 3 show the modal composition of the aplites. Aplite tends to be trimodal: commonly 85-95 per cent is made up of microcline, plagioclase and quartz. The plagioclase seems to be concentrated near the periphery of the main body with a tendency for plagioclase-rich portions to parallel the regional trend (Fig. 4). The same generalizations may apply to the distribution of pyroxene and amphibole (Fig. 5), although the concentrations of these minerals do not always coincide with those of plagioclase. Lack of samples does not permit such generalizations for the satellite bodies. Quartz shows no regularity in distribution.

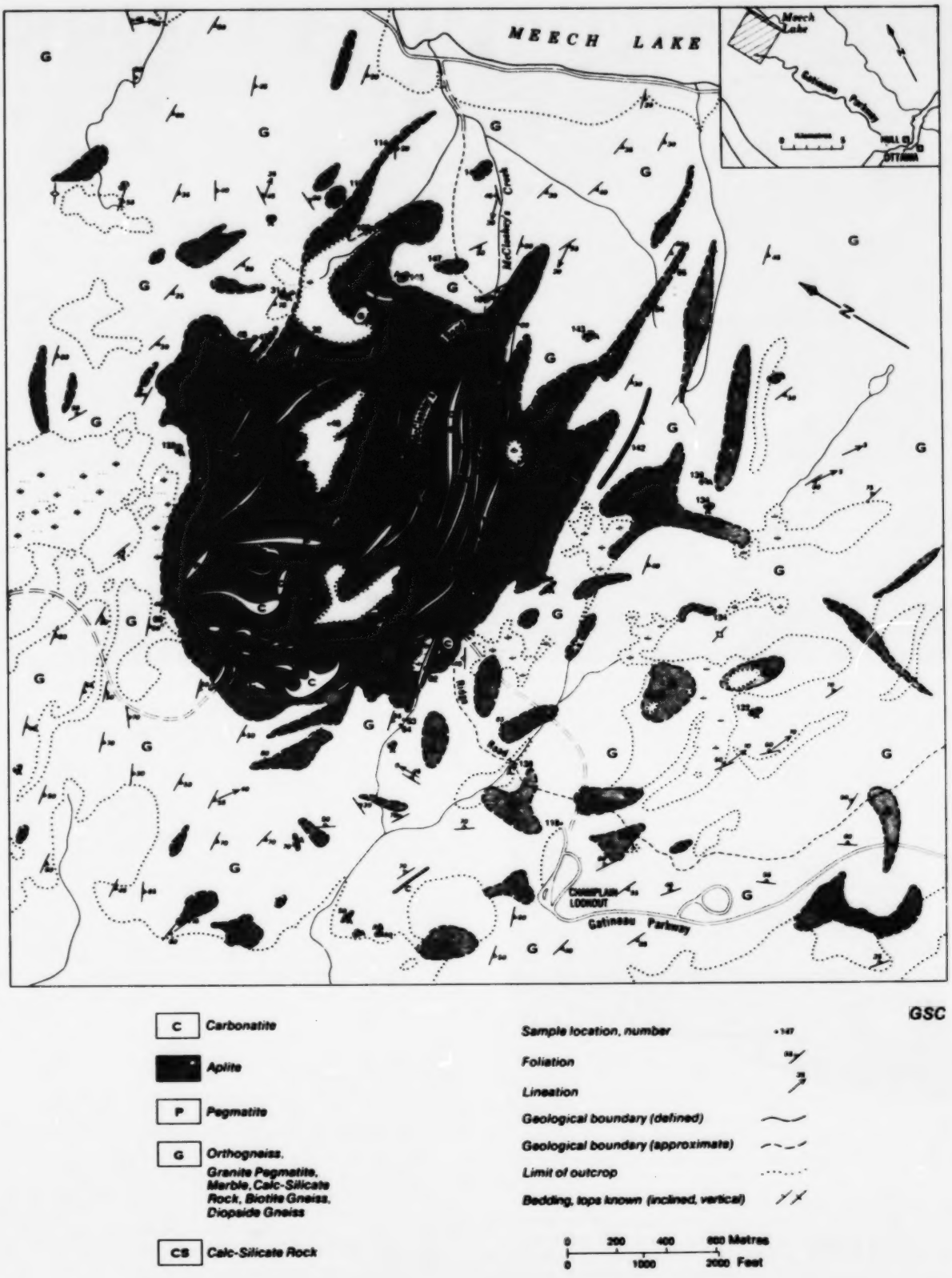

Figure 1. *Geological map of Meech Lake aplite and surrounding area.*

(b)

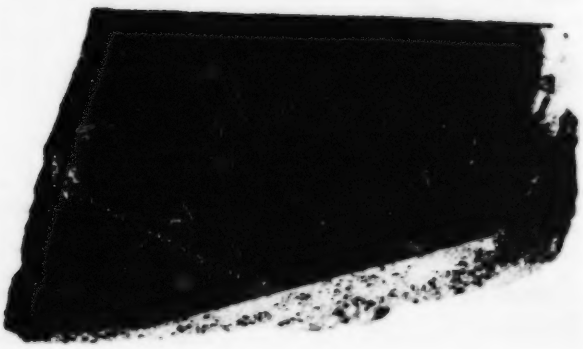

Figure 2. *Contacts of aplite with host rock (syenite gn*
(a) Aplite border at the falls on McCloskey's Creek cl
transects the foliation of the gneiss with a discordanc
about 25°; (b) Sharp contact in sample taken about 1
southwest.

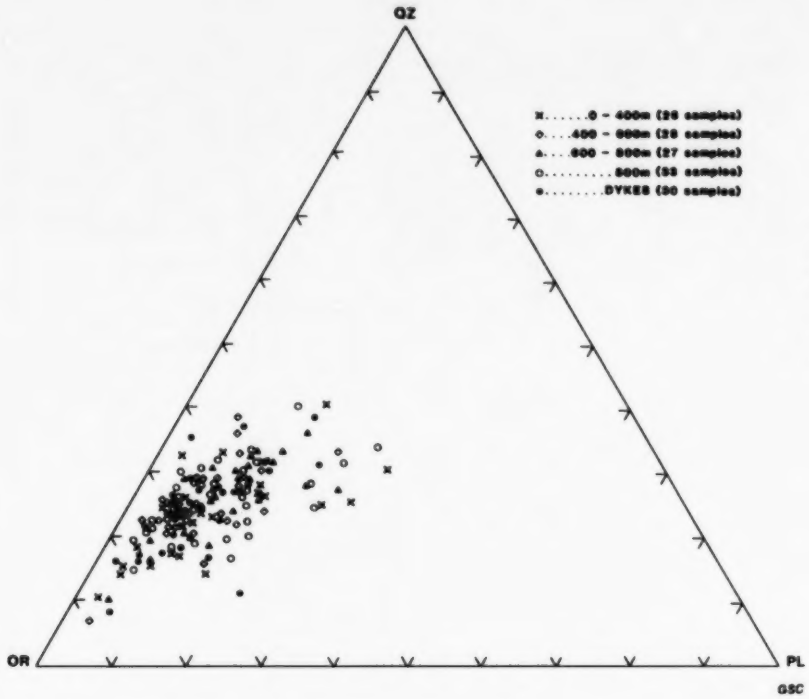

Figure 3

Triangular diagram showing relative proportions of microcline (OR), plagioclase (PL), and quartz (QZ) of 145 aplite samples subdivided according to their distances from the aplite-syenite gneiss contact.

Samples near carbonatites seemingly tend to be somewhat depleted in quartz and enriched in amphibole and pyroxene but the correlation between quartz and pyrobole is not strong. All these conclusions were reached without the aid of statistical tests as a detailed analysis of the modal composition of the stock was not the purpose of this study.

The texture of aplite is saccharoidal and metamorphic fabric is entirely lacking. Orbicular structure, marked by a fringe of radiating sodic amphibole, was described by Béland (1951) and was noted in several places during our study. More commonly, the amphiboles are distributed in planar or curvilinear aggregates of veins (Fig. 6). Under the microscope, grains were seen to be equant and anhedral, with large anhedral feldspars. Some amphibole is euhedral in radiating packets. Pyroxene is commonly corroded and skeletal. Quartz is interstitial in a few samples, more rarely it is graphic or myrmekitic. Most microcline grains are 0.5 to 1.0 mm diameter but both larger and smaller grains are present. A rough banding of coarse- and fine-grained portions was noted in a few outcrops. Quartz, plagioclase and pyroxene tend to be finer grained than microcline; amphibole is commonly coarser. In general, the grain size diminishes on approaching the contact with the orthogneiss host rock.

Figure 7 is a photomicrograph of typical aplite. Microcline gave an optic angle ($2V\alpha$) of 82° (average of 4) with $X' \wedge 001$ in 010 section = 5°, suggesting rather pure microcline in a high state of Al-Si order (Smith 1974, p. 379-383, 387-389). Some grains are cored with plagioclase. Film to braided perthite is visible in many grains.

The plagioclase is albite, its composition in the main body ranges from An_2 to An_8, averaging $An_{4.5}$ (12 optical determinations). Twins are normally albite, more rarely pericline or combined albite-pericline. No regular distribution of compositions with position within the body is evident. Albite from the satellite bodies is similar: composition An_2 to An_7, averaging An_4 (10 optical determinations).

Amphibole is normally sodic (bluish amphiboles with incomplete extinction in 010 sections) but shows a wide range in composition. Twenty-five pyrobole-bearing thin sections from the main body were studied in detail: 9 had amphiboles with Y = b only (magnesioriebeckite orientation), 5 with Z = b only (riebeckite orientation), 6 had grains of both Y = b and Z = b, 1 had an amphibole with optical properties of common hornblende, and 4 showed pyroxenes but no amphiboles. Grains are strongly zoned with the more sodic portions on the peripheries. No regular distribution of amphibole types within the body was observed although sodic-magnesian amphiboles tend to grow in veinlets associated with carbonatites and in the amphibole-phlogopite selvage at the carbonatite aplite contact. Amphiboles from satellite bodies are similar.

Pyroxenes range from almost pure acmite to diopsidic aegirine-augite, with the diopsidic pyroxenes associated with sodic-magnesian amphiboles in veins and in the contact selvage. Acmite is the dominant type within both the main aplite body and satellite bodies.

Biotite-phlogopite is pale to distinct brown. Some grains are partly chloritized. The mineral has the meroxene (Y = b) orientation and normally has the absorption formula $Z = Y > X$, although one thin section showed reverse absorption with $X > Y = Z$, a scheme associated with ferric iron in tetrahedral sites (Hogarth et al., 1970). It should be noted that reversely absorptive phlogopites are not rare in the mica-amphibole selvage at the contact of carbonatites and aplite.

Less common minerals in the aplite include apatite and carbonate (in minor amounts in nearly every thin section), sphene, pyrite, iron oxides, rutile, zircon, a mineral of the pyrochlore group, barite, fluorite, a fibrous zeolite and an unidentified metamict mineral.

Table 1. Modal composition of potassic aplite near Meach Lake, Gatineau Park, Quebec

A	Thin section	Orthoclase	Plagioclase	Quartz	Amphibole	Pyroxene	Opaques	Sphene	Mica	Comments
	1	80.6	4.6	9.9	-	2.7	tr.	tr.	2.0	Ap., Carbonates
	2	63.7	10.2	18.6	-	1.9	0.8	1.8	-	Ap., Carbonates (3%)
	3	67.4	9.7	19.8	-	1.6	0.4	tr.	1.1	Ap., Carbonates
	4	65.4	14.2	14.7	3.7	0.5	0.5	tr.	1.0	Ap., Carbonates
	5	44.6	26.4	26.4	1.5	0.3	0.8	tr.	-	
	6	42.3	25.5	31.0	-	0.8	tr.	0.4	tr.	
	7	48.6	22.4	27.8	0.4		tr.	0.6	0.2	
	8	61.3	7.9	15.7	6.9	4.0	1.1	tr.	3.1	Ap., Carbonates
	9	52.2	14.7	30.7	tr.	1.4	1.0	tr	tr.	
	10	44.2	29.3	25.0	-	0.3	0.6	tr.	0.6	Carbonates
	11	55.2	16.5	25.9	1.0	0.7	0.7	tr.	-	
	12	45.7	23.7	22.7	5.0	2.0	0.3	0.6	tr.	Ap.
	13	56.3	18.7	23.4	1.0	0.2	0.4	tr.	-	Ap., Carbonates
	14	62.6	13.4	23.0	-	0.1	0.7	tr.	0.2	Carbonates
	15	51.8	15.9	23.5	1.0	3.6	1.4	tr.	2.8	Ap., Carbonates
	17	48.2	24.4	23.5	0.4	3.0	0.5	tr.	tr.	
	19	72.5	4.2	17.8	3.7	0.5	0.5	tr.	0.8	Ap., Carbonates
	20	36.3	31.4	29.6	-	0.9	3.8	tr.	tr.	
	21	68.5	10.1	16.3	3.8	tr.	1.0	tr.	0.3	
	22	69.6	6.9	14.8	5.2	2.0	0.4	0.4	0.7	Ap., Carbonates
	23	71.9	9.6	17.4	-	-	0.6	tr.	0.5	Ap.
	24	68.5	15.6	14.1	1.0	6.5	0.5	-	-	
	25	67.2	8.0	21.2	1.0	0.3	1.0	tr.	1.3	Carbonates
	26	53.7	15.8	26.7	0.1	0.1	1.0	tr.	2.6	Ap., Carbonates
	27	63.0	7.3	28.0	-	-	tr.	tr.	1.7	Ap.
	28	42.4	24.1	32.5	-	tr.	0.6	-	0.4	
	29	44.5	18.3	35.6	tr.	0.9	0.7	tr.	-	
	30	52.3	15.0	31.3	-	-	0.4	tr.	1.0	Ap., Carbonates
	32	69.8	5.6	18.1	3.3	1.3	0.5	tr.	1.4	Ap., Carbonates
	33	61.3	16.4	21.6	-	tr.	tr.	-	0.7	
	34	69.7	7.5	20.8	1.0	tr.	0.5	-	0.5	Ap.
	35	39.9	18.4	39.7	-	1.4	0.6	tr.	-	
	36	62.5	12.2	22.4	0.9	tr.	0.5	tr.	1.5	Ap.
	37	67.4	9.3	22.5	-	-	0.3	-	0.5	Carbonates
	38	68.3	6.6	23.2	0.5	-	0.5	-	0.9	Carbonates
	39	66.3	7.8	25.6	-	0.3	tr.	tr.	-	
	40	55.6	8.4	31.6	3.0	0.8	0.6	tr.	-	
	41	85.7	3.2	10.9	-	0.2	tr.	tr.	tr.	Ap.
	42	78.0	4.3	13.7	-	3.1	0.3	0.6	-	Ap.
	43	55.0	17.2	25.7	-	0.5	1.5	0.1	-	Ap.
	44	68.9	6.0	25.1	tr.	tr.	tr.	-	-	
	45	55.6	13.0	29.1	0.5	0.4	1.3	0.1	-	
	46	62.3	8.0	28.4	-	-	0.3	tr.	1.0	
	47	59.8	14.4	23.4	-	0.3	0.1	0.4	1.6	
	48	55.1	13.1	30.3	-	-	0.4	0.3	0.8	
	49	44.1	15.2	39.6	-	-	1.0	tr.	0.1	
	50	65.2	9.3	24.8	0.3	0.4	tr.	tr	tr.	
	51	53.0	13.2	31.9	0.5	0.5	0.6	tr.	0.3	
	52	69.8	5.7	22.0	0.5	1.0	0.6	0.1	0.3	
	53	60.6	10.3	28.7	0.3	-	0.1	-	-	Ap.
	54	63.0	7.3	27.7	tr.	1.5	0.3	tr.	0.2	
	55	61.8	10.6	25.7	-	-	1.0	0.1	0.8	
	56	62.6	10.5	25.8	-	0.5	0.1	tr.	0.5	
	57	59.5	8.6	30.0	-	-	0.7	tr.	1.2	
	58	61.3	10.4	25.5	-	tr.	1.1	0.2	1.5	
	59	65.1	10.4	23.7	-	tr.	0.6	tr.	0.2	
	60	55.8	14.1	27.1	tr.	2.8	tr.	0.1	0.1	Ap., Carbonates
	61	56.8	11.7	29.6	-	1.4	0.4	tr.	0.1	Carbonates
	62	52.5	12.3	32.2	-	-	2.9	tr.	0.1	
	65	68.3	6.3	22.2	-	2.9	tr.	tr.	0.3	
	66	62.4	5.7	27.6	4.3	tr.	tr.	-	-	
	67	68.4	5.0	19.8	3.6	3.0	0.1	0.1	-	Ap., Carbonates
	68	66.7	8.4	22.3	1.0	1.0	0.1	tr.	0.5	
	70	79.9	4.3	15.0	0.4	0.3	0.1	tr.	tr.	Ap.
	71	61.9	3.4	31.7	0.6	2.0	0.3	tr.	0.1	
	72	75.2	3.8	19.0	-	1.5	0.3	tr.	0.2	
	73	57.5	15.6	24.2	2.6	-	0.1	tr.	tr.	
	74	67.9	7.5	21.0	1.6	0.3	0.2	0.3	1.2	
	75	58.2	13.9	26.8	-	0.6	0.4	-	0.1	
	76	68.4	8.0	21.5	1.0	0.4	0.2	0.2	0.3	

A-stock; B-dykes.

(Mineral composition in percent; Ap. is Apatite)

Table 1 (cont.)

Thin section	Orthoclase	Plagioclase	Quartz	Amphibole	Pyroxene	Opaques	Sphene	Mica	Comments
77	64.5	8.7	24.0	1.0	0.5	0.1	0.4	0.8	
78	62.5	11.1	17.3	8.5	0.5	tr.	0.1	-	
79	56.5	13.0	30.0	tr.	-	0.5	-	tr.	Carbonates
80	66.3	5.3	27.2	0.5	0.5	0.2	tr.	-	
81	66.3	8.5	24.5	tr.	tr.	0.3	0.1	0.3	
82	69.4	4.5	21.6	3.2	1.0	0.3	-	-	
84	58.9	8.6	31.5	-	tr.	0.1	-	0.9	
85	64.0	17.4	16.8	-	1.0	0.7	0.1	tr.	
86	50.7	21.7	20.2	0.7	1.8	1.1	tr.	3.8	Ap., Carbonates
87	55.6	11.2	32.2	tr.	0.3	0.3	tr.	0.4	Carbonates
88	69.5	4.9	24.4	-	-	0.6	0.6	-	
89	54.8	13.1	19.5	7.3	2.0	0.8	-	2.5	
90	49.1	16.4	32.2	-	-	1.0	tr.	1.3	
91	61.3	6.5	21.5	5.5	tr.	tr.	tr.	5.2	Ap., Carbonates
92	73.6	3.4	22.0	tr.	0.2	0.7	0.1	tr.	Carbonates
93	58.0	2.5	4.6	24.4	10.0	0.5	-	-	Ap.
94	53.0	12.7	31.1	-	-	0.2	-	3.0	Carbonates
95	62.7	5.8	23.4	0.5	1.2	1.3	0.1	-	Ap., Carbonates (5%)
96	73.4	7.2	15.7	1.2	1.8	tr.	0.7	tr.	Ap., Carbonates
97	73.8	4.7	20.4	0.1	0.2	0.6	tr.	0.2	Ap., Carbonates
98	69.3	5.1	13.5	12.0	0.1	tr.	tr.	-	Ap., Carbonates
99	52.1	15.6	31.3	tr.	-	tr.	tr.	1.0	
100	69.1	4.7	25.5	-	-	0.1	-	0.6	
101	53.5	9.3	35.4	tr.	tr.	0.6	-	1.2	Ap.
102	53.6	15.4	29.5	-	0.6	0.4	0.2	0.3	
103	66.1	7.6	23.8	-	-	1.3	tr.	1.2	Carbonates
104	66.3	10.7	19.6	0.5	1.5	0.8	0.1	0.5	
105	52.4	8.1	37.4	tr.	0.3	0.5	tr.	1.3	
106	62.7	8.4	27.6	0.6	0.2	tr.	-	0.5	
107	61.1	10.2	27.8	-	tr.	0.7	tr.	0.2	
108	61.3	7.1	30.3	-	-	0.1	-	1.2	Ap.
109	64.0	4.8	29.5	-	1.2	0.1	tr.	0.4	Ap.
110	46.9	22.1	27.2	-	0.5	2.5	tr.	0.8	Carbonates
111	48.6	10.3	13.4	18.0	6.0	2.7	1.0	tr.	
112	56.6	13.0	28.6	-	0.7	0.6	0.1	0.4	
113	67.6	4.9	23.8	tr.	1.3	1.2	tr.	1.2	
114	62.9	15.7	19.5	1.4	0.5	tr.	tr.	tr.	
115	59.1	14.9	26.0	-	-	-	-	-	
116	53.8	14.3	31.3	-	-	0.5	-	-	
117	53.4	13.7	24.8	6.3	1.8	-	-	-	
B									
31	57.6	17.6	19.0	2.4	2.0	tr.	-	1.4	
63	66.0	6.3	25.4	-	0.5	1.6	-	0.2	Ap.
64	69.3	6.4	22.6	1.0	-	0.4	0.1	0.2	
118	64.9	14.4	16.0	-	-	1.3	0.2	3.2	Carbonates
119	60.8	11.8	26.8	0.3	tr.	tr.	0.1	0.2	
120	54.7	11.8	19.8	9.8	tr.	0.1	0.4	3.4	
121	65.5	22.2	12.2	tr.	-	0.1	tr.	tr.	
122	57.6	13.3	25.5	0.7	tr.	0.6	0.1	2.2	
123	57.5	14.7	27.6	0.2	tr.	tr.	-	-	
124	69.0	6.2	23.2	0.1	-	0.4	tr.	1.1	
125	65.2	8.2	25.1	0.5	0.2	0.1	0.1	0.6	Ap.
126	65.3	6.3	23.7	tr.	3.0	1.5	tr.	0.2	
127	71.1	3.8	20.7	0.7	3.0	0.4	-	0.3	Ap.
128	67.7	8.0	19.6	4.0	0.4	0.1	tr.	0.2	Ap.
129	62.0	8.3	26.6	0.6	2.0	0.4	tr.	0.1	Ap.
130	52.7	9.3	36.3	-	1.0	tr.	0.1	0.6	
131	65.1	5.3	25.1	3.6	-	0.2	-	0.7	Ap.
132	42.7	18.3	37.9	0.4	0.2	0.4	tr.	0.1	
133	64.5	7.2	15.6	4.0	5.2	1.0	0.1	2.4	
134	61.2	10.4	27.3	tr.	tr.	tr.	tr.	1.1	
135	79.6	2.8	16.0	1.0	0.3	0.2	tr.	0.1	
136	77.3	5.4	17.2	-	0.1	-	-	-	
137	74.9	5.2	7.6	9.3	3.0	-	tr.	tr.	
137A	63.4	12.5	21.9	-	0.2	-	tr.	-	
138	76.6	5.8	16.3	tr.	0.2	0.4	-	0.7	
139	68.5	7.6	22.4	-	0.4	0.9	-	9.2	
140	45.8	22.6	30.7	-	0.3	0.3	0.1	0.2	
140A	66.1	8.3	24.7	-	0.1	0.4	0.1	0.3	
141	67.3	7.1	24.6	-	0.1	0.5	0.1	0.3	
141A	56.1	15.0	28.1	-	0.3	tr.	tr.	0.5	
142	70.1	10.6	18.3	0.1	0.4	0.1	0.1	0.3	
143	52.7	16.0	29.7	0.3	1.0	0.3	tr.	-	
144	58.6	14.3	26.3	-	0.4	-	0.1	0.3	
145	59.8	10.2	25.9	0.6	2.0	1.1	0.1	0.3	
146	63.2	6.8	28.2	1.0	0.7	tr.	-	0.1	
147	59.6	3.3	34.4	1.0	1.7	tr.	tr.	tr.	

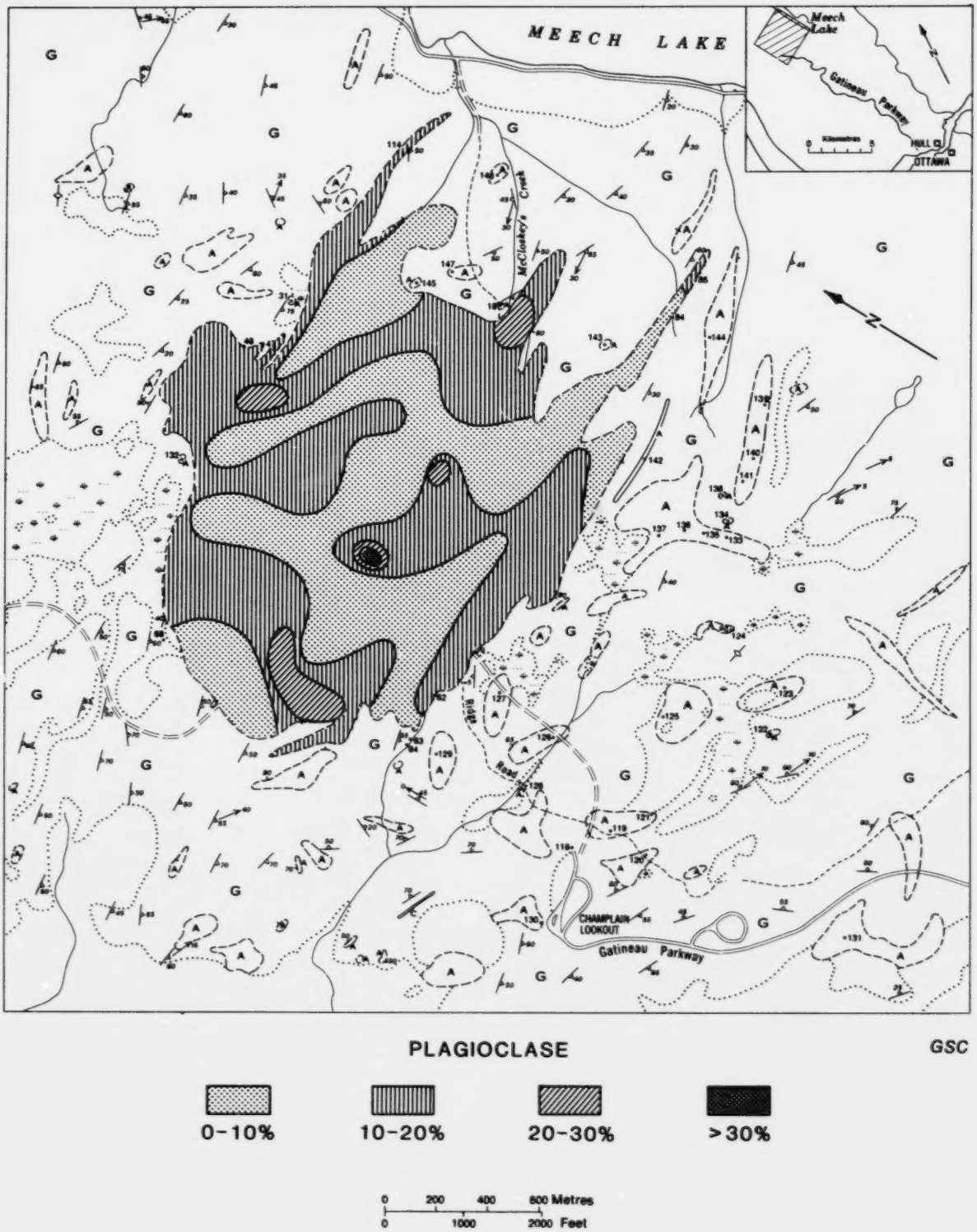

Figure 4. *Schematic contour map of modal percentage of plagioclase in Meech Lake aplite.*

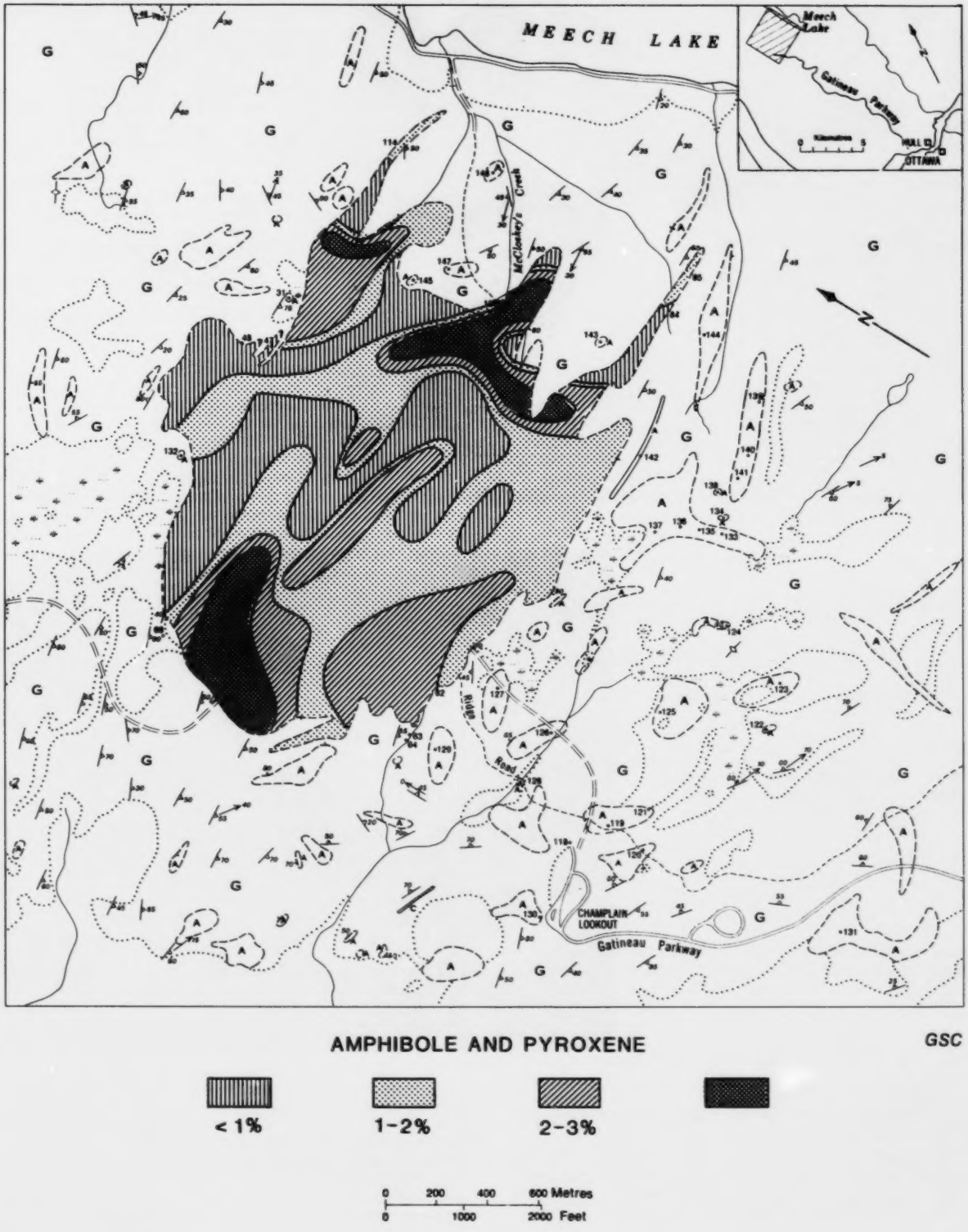

Figure 5. *Schematic contour map of sum of modal percentage values for pyroxene and amphibole in Meech Lake aplite.*

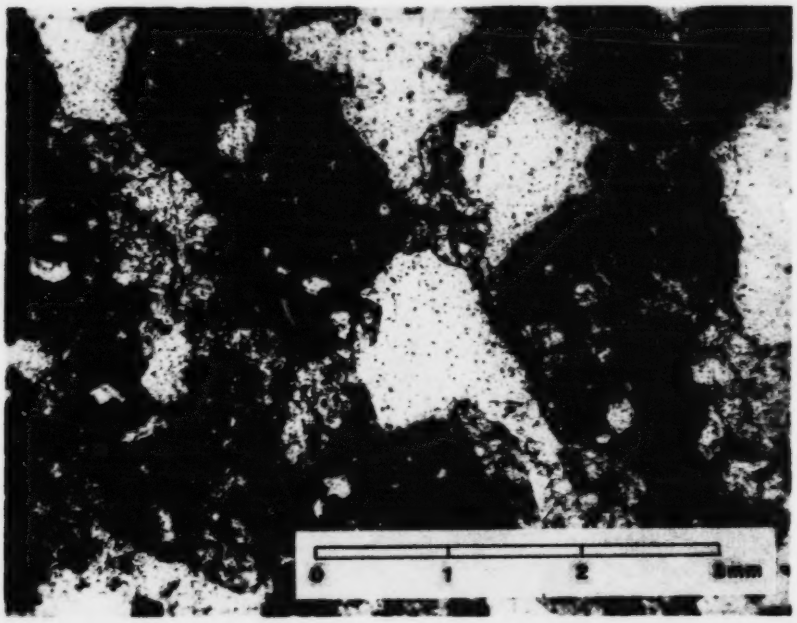

Figure 6

Part of aggregate of amphiboles (dark) in aplite. Sample 7. Nicols not crossed.

Figure 7

Typical Meech Lake aplite texture. Sample 73. Nicols crossed.

Significant field and textural relations are as follows:

(a) Contacts of the principal aplite body are sharp and transect the country rocks.

(b) Dykes and sills of aplite have similar compositions to the principal aplite body which they surround. These dykes and sills have sharp contacts with the country rocks (for example the dyke containing sample 142).

(c) The textures of aplite are not typical of metamorphic rocks.

On the basis of these features we conclude that the Meech Lake aplite has an igneous origin. The aplitic rocks of the stock were generated from magma.

This enables us to develop a stochastic model of crystallization. If this model fits the observations it may be concluded that the aplites were not metasomatically altered. If the model contradicts the observations there are two possible reasons. It may be that the model does not describe the most important features of the crystallization; or it may also be that the rocks were formed as indicated by the model but that they suffered later recrystallization.

If we can prove that the model of crystallization does not contradict the observations we will also be able to investigate the possible influence of the carbonatites on the aplites under study by estimation and interpretation of the parameters of the model. These problems are studied in the following sections of the paper.

Stochastic Model of Aplite Melt Crystallization

The general model for the crystallization of ideal granite developed by Vistelius (1972) is used here with the following simplification. In the general model of ideal granite it is supposed that crystallization of a mineral can be realized in three stages (precotectic, cotectic, and eutectic, respectively) and between any two stages there can be an interruption in the growth of the crystallizing mineral. For example, suppose that a given crystal of plagioclase was generated during the precotectic stage. It grew for some time but the process of growth was discontinued before crystallization reached the cotectic stage. Later some new grains were generated along the cotectic line and their growth was finished before the liquidus reached eutectic composition. Finally there was a separate stage of crystallization at the eutectic point.

Previously Vistelius (1972) used the notation α_{ji} for a grain composed of mineral 1 which was crystallized during stage i. In the present study we observed under the microscope that grains of microcline and albite are rather homogeneous and there are no traces of interruption in crystal growth. Thus it can be assumed that the growth of the crystals, after commencing at a precotectic point, continued without interruption up to the eutectic point. Under these conditions there is no need to use two subscripts for the identification of a grain. Instead of α_{ji} we will use the notation a_i with a single subscript (i) to indicate the composition of the grain. In other words, we assume that there were only three states from the beginning to the end of the process. This is a simplification of the general model for crystallization of ideal granite. However we are allowed to do this in the case of the melt that gave the aplite stock of the present study. All other conditions for crystallization of ideal granite are similar to the conditions assumed before (Vistelius, 1972). The following five conditions can be postulated for the Meech Lake aplite.

Condition 1

The crystallization of the melt extended over the entire volume of the melt. Initially, the melt developed a lattice of nuclei with the intermediate space between the nuclei composed of liquid. In the course of time this lattice was filled up by newly generated nuclei of crystals and, at the end of the process, the entire volume of the melt had been replaced by a solid body composed of different minerals. This process is similar to the filling of a volume in three-dimensional space by means of a succession of lattices superimposed upon one other because each new lattice fills up the intermediate space between crystals generated earlier. Since the melt has a high viscosity and the specific gravities of microcline, albite, and quartz are about the same, we believe that there was no mutual displacement of the growing crystals of microcline, albite and quartz. In other words, it is likely that there was no gravitational differentiation in the investigated aplite stock for individual crystals of quartz, albite and microcline.

If an arbitrary straight line is constructed through a rock generated when condition (1) is fulfilled we would obtain along this line a sequence of grains of rock-forming minerals whose relative positions are determined by the primary positions of their nuclei in the melt. Let us denote a separate grain as 'a' with subscript 'i' so that a_i indicates the grain of composition i, and superscript (h) so that $a_i^{(h)}$ indicates the position of the grain in the sequence. Then the expression $a_i^{(h)}$ i ε {Or, Q, Ab} means that in some sequence there is a grain 'a' which is composed of K-Na feldspar (Or), quartz (Q), or albite (Ab), and occupies position h in the sequence. A complete sequence will be represented as

$$\ldots, a_i^{(h)}, \ldots$$

or, in abbreviated form, as $\{a_i^{(h)}\}$

If there is a sequence

$$\ldots, a_i^{(h)}, a_j^{(h+1)}, \ldots$$

and we are able to predict the occurrence of a_j in position h+1 not with certainty but with some probability, we will refer to this sequence as a random sequence. The sequences of grains along the arbitrary straight lines mentioned before are typical random sequences. There are different models of random sequences but their characteristic feature is prediction of the occurrence of a grain in a specific position with a probability P that, in general, satisfies $0<P<1$. The probability of appearance of the grain a with composition i in position h is indicated as $P(a_i^{(h)})$; i ε {Or, Q, Ab}. We refer to i as a state originating from the set {Or, Q, Ab} with elements Or, Q, and Ab.

Condition 2

Under the conditions indicated, the specimen being studied in a thin section represents an isotropic rock. An infinitely long sequence of grains in such a rock along a straight line then has the following property which should be included in the formal model. Let us look at the combination of grains which occupy positions $h_1, h_2, \ldots, h_n$ (these positions are not necessarily juxtaposed), and have compositions i, j, . . ., k. The probability of the appearence of such a combination is

$$P(a_i^{(h_1)}, a_j^{(h_2)}, \ldots, a_k^{(h_n)}).$$

Let us also consider the probability of this combination appearing after some shift ℓ:

$$P(a_i^{(h_1+\ell)}, a_j^{(h_2+\ell)}, \ldots, a_k^{(h_n+\ell)}).$$

Then the isotropy condition implies that in our model

$$P(a_i^{(h_1)}, a_j^{(h_2)}, \ldots, a_k^{(h_n)}) = P(a_i^{(h_1+\ell)}, a_j^{(h_2+\ell)}, \ldots, a_k^{(h_n+\ell)}) \quad (1)$$

for any integer n, any set of states i, j, . . ., k, and any $h_1, h_2, \ldots, h_n$.

This property is referred to as stationarity of the random sequence. In particular, for h=1

$$P(a_i^{(h_1)}) = P(a_i^{(h_2)}) = \ldots = P(a_i). \quad (2)$$

$P(a_i)$ is referred to as the stationary probability of the state of i. A set of stationary probabilities $P(a_i)$ can be written in the form of a column vector P_{st}.

Condition 3

As mentioned in condition (2), the isotropic arrangement of grains in a granite places restrictions on the distribution of grains along arbitrary straight lines which cut the rock. In particular all probability properties of the sequence should be independent of the direction of movement along the sequence. This property of a sequence is referred to as reversibility. This is a special feature of the isotropic rock which is implied by the assumption of stationarity of a sequence discussed in condition 2. For reversible sequences, we have

$$P(a_i^{(h)}, a_j^{(h+1)}) = P(a_j^{(h)}, a_i^{(h+1)}). \quad (3)$$

Condition 4

Let us now derive a condition from some basic assumptions regarding the growth of an individual nucleus to a crystal. This growth proceeded by the accumulation of material drawn from the surrounding medium (the aplite melt). Since this process took place in a closed system without the introduction of additional material, the medium (aplite melt) in the vicinity of the growing crystal was depleted in material with the composition of the crystal. However, if the process was proceeding in this way it is obvious that, next to a growing crystal of some specific composition, a new nucleus of a crystal of the same composition is unlikely to be generated. Suppose, for example, that a nucleus of quartz was growing from the aplite melt by accumulation of SiO_2. Then the vicinity of the growing quartz crystal would contain smaller concentrations of SiO_2 than the original melt. This is the reason that the growing crystal of quartz had a low probability of getting another crystal of quartz of the same generation in direct contact with it. This argument applies only to crystallization in closed systems where all crystals were generated during a single stage of crystallization.

Let us now consider the dimension of the zone of depletion in concentration of some of its components around the growing crystal. Definitive data on this subject are not in existence but it would seem that this zone is rather narrow, probably not wider than the average grain size. Thus the zone around the growing crystal, where the occurrence of a crystal of the same composition has a small probability, is probably not wider than the size of a single grain. If this reflects what actually happened in the crystallizing melt, then each random sequence of grains along a straight line through the aplite can be described as follows:

If we take the probability of the combination $P(a_i^{(h)}, a_j^{(h+1)})$ and divide it by $P(a_i^{(h)})$, (assuming that $P(a_i^{(h)}) > 0$) we obtain

$$P(a_j^{(h+1)} \mid a_i^{(h)}) \tag{4}$$

which is referred to as a transition probability or a probability of transition through state a_i to state a_j in one step. Accordingly the division of the probability $P(a_i^{(h-1)}, a_j^{(h)}, a_k^{(h+1)})$ by the probability $P(a_i^{(h-1)}, a_j^{(h)})$ can be written as

$$P(a_k^{(h+1)} \mid a_j^{(h)}, a_i^{(h-1)})$$

and referred to as the probability of transition through a_i and a_j to a_k.

Let us now consider an arbitrary event which is dependent only upon the appearance of grains in positions of the sequence with numbers smaller than h. This event can be written as A_{h-1}. Similar events such as A_{h-2} also can be defined.

Division of the joint probability $P(A_{h-1}, a_i^{(h)}, a_j^{(h+1)})$ by the probability $P(A_{h-1}, a_i^{(h)})$ then gives the transition probability $P(a_j^{(h+1)} \mid a_i^{(h)}, A_{h-1})$.

If our petrological considerations concerning crystallization were correct, the equality

$$P(a_j^{(h+1)} \mid a_i^{(h)}, A_{h-1}) = P(a_j^{(h+1)} \mid a_i^{(h)}) \tag{5}$$

should be fulfilled for all i and j, any A_{h-1} and for any position h. A random sequence with condition (5) is referred to as a simple (first order) Markov chain (or sequence). If

$$P(a_j^{(h+1)} \mid a_i^{(h)}, a_k^{(h-1)}, A_{h-2}) = P(a_i^{(h+1)} \mid a_i^{(h)}, a_k^{(h-1)}) \tag{6}$$

for the random sequence, this sequence is referred to as a second order Markov chain. Correspondingly, a sequence with the property $P(a_i^{(h)} \mid A_{h-1}) = P(a_i^{(h)})$ is referred to as a zero order Markov chain or Bernoullian sequence (= sequence of independent events)

A simple Markov chain with condition (4) which is independent of h is referred to as homogeneous. If it is known that the sequence is a simple Markov chain, and this sequence possesses the property $P_0 = P_{st}$ and is homogeneous, it is called a stationary Markov chain.

If the transition probabilities of a simple Markov chain and its reverse sequence are equal, the sequence is referred to as a reversible Markov chain. The reverse of a simple, stationary sequence Markov chain is a simple, stationary Markov chain also. The interrelation

$$P(a_i \mid a_j) = P(a_j \mid a_i)\, P(a_j) / P(a_i) \tag{7}$$

holds true for reversible simple, stationary Markov chains. This represents a particular case of a reversible random sequence (2).

Condition 5

In the previous section, some characteristics of the crystallization process were modelled by using the Markov property. The resulting Markov chain should have transition probabilities on the main diagonal of the matrix for the chain which are less than the stationary probabilities of the same states. These diagonal elements can be used for estimation of the properties of the crystallizing melt.

At this point we can summarize the model as follows. It entails five separate interrelations which we should check in our observations

$$P_0 = P_{st} \tag{8a}$$

$$P(a_i^{(h)} \mid a_j^{(h-1)}) = P(a_i^{(h+k)} \mid a_j^{(h+k-1)}) \text{ for any h and k} \tag{8b}$$

$$P(a_i^{(h)} \mid a_j^{(h-1)}) = P(a_i^{(h)} \mid a_j^{(h-1)}, A_{(h-2)}) \tag{8c}$$

$$P(a_i \mid a_j) = P(a_j \mid a_i)\, P(a_i)/P(a_j) \tag{8d}$$

$$0 < c_{ii} < 1 \tag{8e}$$

where c_{ii} is from

$$P(a_i \mid a_j) = c_{ji} P(a_i)$$

since it is possible to prove that the random sequence with properties (8a to 8d) of the model permits the replacement of $P(a_i \mid a_j)$ by $c_{ji}P(a_i)$ with $c_{ij} = c_{ji} > 0$. In matrix form our model can be rewritten as

$$P_{i;j} = C\, \Lambda_p \tag{9}$$

with

$$C = \begin{bmatrix} c_{ii} & c_{ij} & c_{ik} \\ c_{ij} & c_{jj} & c_{jk} \\ c_{ik} & c_{jk} & c_{kk} \end{bmatrix} \quad ; \quad \Lambda_p = \begin{bmatrix} P(a_i) & 0 & 0 \\ 0 & P(a_j) & 0 \\ 0 & 0 & P(a_k) \end{bmatrix}$$

where $P(a_i)$, $P(a_j)$, and $P(a_k)$ are stationary probabilities of the states i, j, and k, (i, j, k ε { Or, Q, Ab } in our case), and where $c_{ij} = c_{ji} > 0$ and $0 < c_{ii} < 1$.

Now we should develop a method to determine if our model is in agreement with observations in thin sections or that the investigated stock possessed the features which are typical for a body generated directly from a melt. This method will be based on the theory of mathematical statistics.

STATISTICAL TESTING OF THE MODEL

The previous model describes our initial hypotheses on the genesis of the aplites. Now we should develop statistical hypotheses which correspond to the model and which can be tested against the observations. These hypotheses should include the checking of homogeneity, Markov properties, and the conditions for the set of parameters c_{ij}. In principle, it is desirable to test all statistical hypotheses simultaneously but as there is no method for this at present, we must check them step by step, taking each hypothesis separately.

Our random sequences are composed of the outcomes of trials. Three outcomes are possible: an arbitrary grain can be a grain of microcline, a grain of quartz, or a grain of albite. We denote microcline as 1, quartz as 2, and albite as 3. Each of these figures will be referred to as a state. Thus we have random sequences of the three states {1,2,3}. Each specimen investigated for this study was characterized by about 1000 grains counted along horizontal lines under the microscope. The specimens contain not only the minerals which we are investigating but also amphiboles and other minerals. A thousand grains arrranged along one line with direct contacts between all grains would occupy a distance greater than the width of the thin section. Moreover, there are holes and other defects in thin sections. For these reasons, it is not possible to get a sequence of 1000 grains without interruptions. However, a composite sequence of about 1000 grains can be constructed from a number of subsequences. If we write a grain of composition i in position h as $a_i^{(h)}$, the grains in positions h-1 and h+1 have direct contacts with the grain $a_i^{(h)}$. Within each subsequence all grains have direct contacts with their neighbouring grains except the grains at the beginning and the end of the subsequences. As a rule, subsequences of grains should contain not less than 10 grains. A sequence is composed of subsequences as follows:

$$\ldots,1,1,3,1,2,2,3,1,1,2;\ 3,3,2,1,3,3,2,1,2,1,2,2,2,\ldots \qquad (10)$$

where ';' indicates the end of a subsequence. Figure 8 shows part of a sequence which is typical for the Meach Lake aplite stock. We will now test a number of statistical hypotheses concerning sequences of this type.

Hypothesis of Homogeneity

We start with the construction of a frequency distribution for each mineral i, considering the transition from an observed mineral i to all minerals (Or, Q, Ab), from pairs of adjoining minerals ij to all minerals, and from triplets ijk to all minerals composing the sequence. Thus we subsequently obtain matrices N_i (one-dimensional), $N_{i;j}$, $N_{i,j;k}$, and $N_{i,j,k;l}$ for i,j,k,l ε {Or, Q, Ab}. In the last three

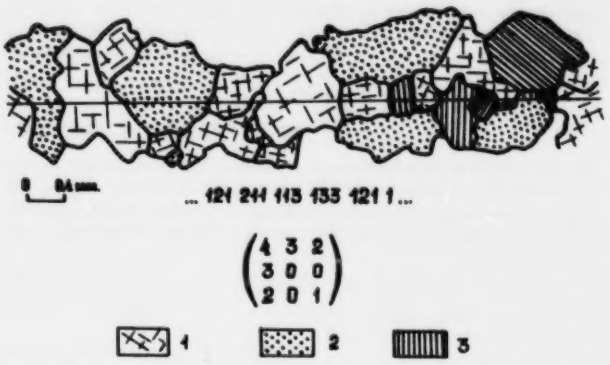

Figure 8. *Typical subsequence of grains in Meech Lake aplite (from sample 39). 1. Microcline; 2. Quartz; and 3. Albite. Sequence of numbers below picture indicates the successive transitions counted. Also shown is the matrix of transition frequencies for this subsequence.*

cases we have matrices with sizes 3x3, 9x3 and 27x3, respectively. These matrices are obtained for every composite sequence. At the same time, we obtain similar matrices for each subsequence separately to compare the frequencies of the matrices for each subsequence with the frequencies of the matrix for the composite sequence which is used as the 'theoretical' or model sequence for the subsequences. The corresponding chi-squared values satisfy:

$$\chi^2(h) = \sum_{i,j} \frac{\left(n_i \cdot n_{ij}^{(h)} - \frac{n_{ij} \cdot n_i^{(h)}}{n_i} \right)^2}{n_i^{(h)} \cdot n_{ij}} \qquad (11)$$

where n_i is the number of grains a_i in the entire sequence, n_{ij} is the number of juxtaposed pairs of grains $(a_i^{(\ell)}, a_j^{(\ell+1)})$ in the entire sequence, with $n_i^{(h)}$ and $n_{ij}^{(h)}$ representing these numbers for the h-th subsequence. Separate values obtained by equation (11) can be added to give

$$\chi^2 = \sum_{h=1}^{k} \chi^2_h \qquad (12)$$

The number of degrees of freedom (ν) for three minerals (or states) is equal to

$$\nu = 3^m \times 2(k-1) \qquad (13)$$

where m represents the order of the Markov chain for which homogeneity is being tested, and k is the number of subsequences in the composite sequence. After a sequence has been shown to be homogeneous we can determine its Markov order.

Markov Hypotheses

Two Markov hypotheses are compared with each other by means of the statistic or likelihood ratio Λ. As we will see later, the investigation of the Meech Lake aplites requires the testing of only two pairs of alternative hypotheses, (1) Bernoullian sequence against simple Markov chain (H_0:H_1), and (2) simple Markov chain against second order Markov chain (H_1:H_2).

The hypothesis H_0 against H_1 is tested by the value of the statistic

$$\Lambda_{H_0:H_1} = 2 \sum_{i,j} f_{i;j} \ln \frac{N f_{i;j}}{f_{\cdot;j} f_{i;\cdot}} \qquad (14)$$

and the hypothesis H_1 against H_2 by

$$\Lambda_{H_1:H_2} = 2 \sum_{i,j,\ell} \bar{n}_{i,j;\ell} \ln \frac{\bar{n}_{\cdot j;\cdot} \bar{n}_{i,j,\ell}}{\bar{n}_{\cdot j;\ell} \bar{n}_{i,j;\cdot}} \qquad (15)$$

where '.' indicates addition of all values for a subscript in this position and $\bar{n}$ indicates an observed frequency value. A similar test for $H_2:H_3$, which was described by Vistelius et al. (1972), has been incorporated in the computer program of Appendix 1.

If H_{m-1} holds true, Λ with m = 1,2,. . . for $H_{m-1}:H_m$ is asymptotically distributed as central χ^2 with number of degrees of freedom equal to

$$\nu = s^{(m-1)} (s-1)^2 \qquad (16)$$

where s represents number of states (minerals) in the chain (in our case, s = 3).

The tests of homogeneity and Markov hypotheses are performed in the first part of the computer program of Appendix 1. Once we have determined the Markov order we can take the next step which is performed in the second part of the program of Appendix 1. At this step we deal with the estimation of the parameters c_{ij} and c_{ii} from the observations.

The Parameters cij

These parameters can be estimated on the basis of equation (8e) using the method of minimum χ^2. In our case, we use the matrix $P_{i;j}$ which is found by observation, viz. direct counting of the numbers of contacts between grains of Or, Q, and Ab along a straight line across the thin section under the microscope. From the observations we also have the matrix Λ_p. We assume that the matrices $P_{i;j}$ and Λ_p reflect real interrelations. Then we can estimate the elements of the matrix C in equation (9). The minimum χ^2 method gives values of c_{ij} assuming that the condition $c_{ij} = c_{ji}$ holds true (Vistelius et al., 1972). The computer program in Appendix 1 contains the formulae for these estimators of c_{ij}. This algorithm has been documented in more detail in Vistelius et al. (1972). By means of it we can find the minimum χ^2 for a homogeneous reversible Markov chain of the first order which corresponds to our model.

Comparison of the observed matrix of a chain with a computed matrix which satisfies interrelation (9) should be performed with a single degree of freedom. Thus if comparison of the matrices gives a value of χ^2 which is greater than 3.84 (χ^2 = 3.84 for 5% level of significance), our model should be rejected.

The last step in checking the model consists of estimation of the interrelations between $P(a_i|a_i)$ and $P(a_i)$. In other words, we should check the inequality $0 < c_{ii} < 1$ which is equivalent to $P(a_i|a_i) < P(a_i)$. No special statistical tests exist to compare these frequencies and, therefore, we have used approximate formulae which follow from the general theory of binomial frequencies. These formulae are:

$$\underline{P}_{i\alpha} \approx \frac{X(\alpha, 2n_i)}{\frac{1}{2}X(\alpha, 2n_i) + 2N - n_i - 1} \qquad (17a)$$

$$\bar{P}_{i\alpha} \approx \frac{X(1-\alpha, 2n_i+2)}{\frac{1}{2}X(1-\alpha, 2n_i+2) + 2N - n_i} \qquad (17b)$$

$$\underline{P}_{ii\alpha} \approx \frac{X(\alpha, 2n_{ii})}{\frac{1}{2}X(\alpha, 2n_{ii}) + 2n_i - n_{ii} - 1} \qquad (17c)$$

$$\bar{P}_{ii\alpha} \approx \frac{X(1-\alpha, 2n_{ii}+2)}{\frac{1}{2}X(1-\alpha, 2n_{ii}+2) + 2n_i - n_{ii}} \qquad (17d)$$

In these expressions, $\underline{P}$ indicates the lower boundary of a confidence interval, and $\bar{P}$ indicates its upper boundary with α for level of significance; $X(\alpha, m)$ is a fractile of the central χ^2 distribution with m degrees of freedom. Linear interpolation of the values of $X(\alpha, m)$ using Owen's (1962) tables for α equal to 0.01 or 0.05, $1 \leq n_i \leq 1000$, and $1 \leq n_{ii} \leq 1000$ gives a precision of approximately 0.01. When the number of degrees of freedom exceeds 100, the Wilson-Hilferty approximation (Johnson and Kotz, 1970, p. 176) can be used also.

Equation (17) permits the estimation of approximate confidence intervals for the estimated frequencies and in this way we can check to what extent the relationship (8e) is fulfilled. For interpretation of the results of application of equation (17) it is convenient to use a special type of diagram which is explained in the next section.

Our results can be summarized as follows. When we obtain a sequence of grains by observation and want to check the model of ideal granite for this sequence we should perform the following operations:

(a) Homogeneity Test

If we prove statistically that the sequence is homogeneous we can continue the testing of our model. On the other hand, if the hypothesis of homogeneity is rejected, the model of ideal granite is rejected.

(b) Test of Markov Order of the Sequence

If we obtain acceptable results from $H_0:H_1$ for H_0, and from $H_1:H_2$ for H_1, we continue our investigation. If H_1 is rejected from $H_1:H_2$, the model of ideal granite should be rejected.

(c) Calculation of Conditional Minimum χ^2

We must calculate the conditional minimum χ^2 with $c_{ij} = c_{ji}$; $c_{ii} > 0$ for comparison of the observed matrix with a matrix of calculated values satisfying these conditions. If we get $\chi^2 > 3.84$, we continue our investigation. If $\chi^2 < 3.84$, the model of ideal granite should be rejected.

(d) Use of Equation (17) and Comparison of the Relative Positions of the Confidence Intervals for $P(a_i|a_i)$ and $P(a_i)$

If for each state i the interval for $P(a_i)$ is located to the right of a similar interval for $P(a_i|a_i)$, it can be assumed that the rock is an unaltered ideal granite. This test is unstable because secondary alterations can change $P(a_i|a_i)$ rather easily.

If our rock was crystallized from a melt, viz. if it is a granite of magmatic origin, and if this assumption is supported by geological field evidence, we can conclude that the passing of tests (a) to (d) proves with a high probability that the primary magmatic granite was not modified significantly by the development of secondary microcline (Or), quartz (Q) or albite (Ab). In general, we are not able to infer much about the origin of the granite if we investigate specimens only. It is also necessary to know something of the geological setting of the stock. Only if the regional setting indicates that the granite is magmatic can we use the model of the ideal granite. If in this situation the model of the ideal granite is consistent with the observations we can state that the magmatic granite was not altered after crystallization.

Table 2. Data block and some results for sample 25. A-data block; B-some results obtained by the computer program in Appendix 1. See text for explanation

A.

32122312312131313123131313131312313131212132122122231321212213212;311113232131312211212131212131311311211312231212333321312112321121112112131212121312121122123212121313;121313113213212;212131213121212132121212123112131321212131321321321321212132322213131212121323112131313212121313113121121312131;121313112113123212;111312121221312121331212133131212112122121312323131312132222131212121213121212321313212211213212121213112132;132122121112121222312121212121312131213212131213121321212121213121222322121312131321212131312112121212131211212213212121113312111321212121312;131312131313131232132122121312312131213212121312323231211121222121231212121131232121312111321212213121312121312312121111232122131213112212211211123122123312111232121312131121222131331312132123121312121312323231211121222121312131312121312121312121

Table 3. Sequences of grains of orthoclase, quartz and plagioclase similar to first-order Markov chains (ideal granites). A-stock; B-dykes

			A for Markov chain order alternatives		χ^2-test for homogeneity			
Specimen number	Length of sequence (in number of grains)	Number of subsequences	H_0:H_1 $\chi_{0.05} = 9.49$	H_1:H_2 $\chi_{0.05} = 21.03$	Number of degrees of freedom	Theoretical $\chi^2_{0.05}$	Observed χ^2	χ^2-test for ideal granite matrix $\chi^2_{0.05} = 3.84$
A								
2	1125	15	180.28	16.44	84	106.40	71.33	0.06
4	1128	20	233.41	15.60	114	139.92	119.92	1.01
5	1123	9	223.17	11.92	48	65.17	50.16	0.48
6	1094	10	195.41	19.01	54	72.15	67.30	0.95
7	1168	9	321.75	6.85	48	65.17	67.76	2.72
8	1080	31	152.27	14.17	180	212.30	180.82	0.98
9	1145	10	165.71	14.63	54	72.15	49.15	0.29
12	1051	16	151.21	9.56	90	113.14	93.00	0.65
13	1177	8	176.16	13.32	42	58.12	37.27	0.17
14	1128	7	377.33	18.21	36	51.00	43.12	1.88
15	1047	23	127.17	17.86	132	159.82	132.71	0.88
19	1134	19	252.83	16.99	108	133.26	130.61	2.09
20	1108	8	87.36	17.23	42	58.12	45.00	1.93
21	1101	10	152.27	17.48	54	72.15	50.73	2.20
22	1094	28	105.18	10.20	162	192.17	152.70	0.65
23	1141	8	259.14	9.23	42	58.12	37.53	0.34
24	1125	8	247.87	14.86	42	58.12	34.34	2.56
26	1093	15	65.85	9.50	84	106.40	100.08	2.56
27	1116	11	83.08	19.82	60	79.08	86.50	0.02
32	1201	24	192.63	16.07	138	166.42	135.30	1.98
33	1106	10	135.98	20.44	54	72.15	48.20	0.38
34	1121	11	270.75	20.14	60	79.08	78.10	0.10
35	1149	6	200.55	8.72	30	43.77	32.37	0.53
38	1128	8	255.81	16.38	42	58.12	51.56	0.14
39	1116	10	237.32	9.64	54	72.15	48.37	0.55
40	1127	8	315.65	14.65	42	58.12	57.24	0.63
41	1140	9	239.16	6.48	48	65.17	43.94	0.41
42	1109	9	186.65	11.71	48	65.17	55.20	0.48
43	1093	8	166.62	11.98	42	58.12	59.68	0.03
46	1106	11	165.15	13.64	60	79.08	72.84	1.52
48	1104	8	77.32	8.80	42	58.12	31.02	1.65
50	1135	7	80.32	16.58	36	51.00	36.97	3.35
51	1136	7	101.45	16.64	36	51.00	24.70	1.74
53	1127	8	207.62	17.70	42	58.12	45.32	0.36
54	1121	9	328.86	5.58	48	65.17	53.96	1.61
55	1073	9	222.34	18.65	48	65.17	40.77	0.22
56	1115	10	139.02	15.73	54	72.15	54.83	0.52
57	1106	12	115.01	8.61	66	85.97	61.21	1.69
59	1084	9	227.52	11.61	48	65.17	37.46	0.86
60	1121	10	142.23	7.53	54	72.15	54.03	0.07
61	1145	8	297.94	10.09	42	58.12	34.68	1.93
62	1088	6	136.76	11.23	30	43.77	30.87	0.08
65	1079	12	140.67	9.02	66	85.97	70.04	1.43
66	1098	9	288.14	11.03	48	65.17	68.76	1.87
67	1072	12	313.17	11.84	66	85.97	51.87	0.17
68	1109	7	290.76	14.27	36	51.00	33.07	0.36
70	1087	9	269.80	11.29	48	65.17	72.17	2.58
71	1084	10	140.74	13.73	54	72.15	59.11	0.03
72	1065	6	191.40	9.74	30	43.77	38.42	1.94
74	1061	8	166.26	10.00	42	58.12	48.04	0.74
75	1064	7	266.25	17.67	36	51.00	52.24	0.02
76	1086	8	87.07	20.01	42	58.12	43.21	0.10
77	1042	9	205.85	5.22	48	65.17	46.07	1.03
78	1025	9	326.57	13.33	48	65.17	51.03	0.18
79	1058	11	382.82	13.07	60	79.08	73.20	0.03
80	1040	8	132.04	4.72	42	58.12	36.09	0.77
81	1037	8	351.28	15.42	42	58.12	31.34	0.13
82	1046	12	115.03	8.31	66	85.97	66.54	0.33

Table 3 (cont.)

Specimen Number	Length of sequence (in number of grains)	Number of subsequences	A for Markov chain order alternatives		χ^2-test for homogeneity			χ^2-test for ideal granite matrix
			H_0:H_1 $\chi_{0.05} = 9.49$	H_1:H_2 $\chi_{0.05} = 21.03$	Number of degrees of freedom	Theoretical $\chi^2_{0.05}$	Observed χ^2	$\chi^2_{0.05} = 3.84$
A								
84	1055	9	237.25	6.47	48	65.17	39.02	0.08
85	1037	8	371.76	15.16	42	58.12	50.91	2.08
86	1084	36	115.25	12.86	210	244.80	189.69	3.05
88	1060	10	134.96	16.48	54	72.15	51.32	2.15
89	1106	20	60.09	15.65	114	139.92	111.50	1.51
90	1073	13	188.22	11.36	72	92.81	90.83	0.06
91	1076	32	142.74	9.93	186	218.80	164.00	0.31
92	1077	10	217.61	7.25	54	72.15	41.15	1.24
93	1071	54	51.61	16.60	318	360.50	282.10	0.53
94	1144	9	203.99	15.43	48	65.17	55.55	0.04
95	1090	24	142.99	7.30	138	166.42	130.34	0.21
97	1085	14	224.04	12.87	78	99.62	94.20	0.51
98	1103	14	239.67	13.16	78	99.62	74.33	0.04
99	1098	11	148.36	5.70	60	79.08	68.76	3.47
100	1128	8	244.45	18.39	42	58.12	55.02	2.41
101	1103	7	145.83	12.76	36	51.00	60.23	2.31
102	1159	8	188.91	15.38	42	58.12	57.58	1.11
103	1090	14	192.09	12.57	78	99.62	115.51	0.19
104	1127	15	217.39	8.73	84	106.40	74.46	0.81
105	1122	9	148.07	14.01	48	65.17	53.35	0.55
106	1127	10	218.32	4.74	54	72.15	83.15	0.03
107	1133	10	167.07	13.05	54	72.15	61.40	0.88
108	1113	11	182.50	13.75	60	79.08	67.16	2.21
109	1127	10	406.53	8.60	54	72.15	56.21	0.30
110	1103	7	311.29	4.46	36	51.00	41.99	1.78
111	1129	28	39.33	14.23	162	192.17	161.87	1.30
113	1131	9	170.13	8.45	48	65.17	33.41	0.63
114	1069	14	126.85	12.66	78	99.62	101.39	0.43
116	1070	15	210.70	11.61	84	106.40	90.39	0.22
117	1082	16	188.16	15.42	90	113.14	128.89	0.95
B								
63	1073	11	128.61	11.57	60	79.08	84.59	0.16
64	1105	8	195.80	6.45	42	58.12	43.50	0.79
118	1094	19	160.39	12.13	108	133.26	121.00	3.29
119	1113	13	131.40	12.05	72	92.81	69.37	1.35
120	1130	6	189.02	12.26	30	43.77	34.18	3.16
122	1099	8	63.35	7.30	42	58.12	52.23	0.77
124	1073	7	217.70	12.41	36	51.00	31.62	0.17
125	1069	9	263.76	16.87	48	65.17	57.72	0.05
126	1089	6	105.50	11.24	30	43.77	35.53	0.15
127	1079	10	111.69	17.63	54	72.15	54.51	2.03
128	1097	8	126.66	17.95	42	58.12	54.57	0.82
129	1089	8	222.18	8.04	42	58.12	39.64	0.34
130	1093	7	124.46	17.80	36	51.00	38.35	0.14
131	1079	11	69.94	11.76	60	79.08	57.21	0.07
133	1053	12	199.95	9.89	66	85.97	47.42	3.48
134	1121	10	177.11	14.50	54	72.15	64.36	0.58
135	1049	8	162.35	5.68	42	58.12	26.52	0.07
136	1031	11	99.15	21.00	60	79.08	88.68	0.17
137	1023	28	126.81	12.62	162	192.17	137.38	1.72
137A	1073	6	246.15	7.23	30	43.77	16.10	1.07
138	1068	6	169.45	11.99	30	43.77	16.58	0.53
139	1072	9	105.34	20.89	48	65.17	44.74	0.52
140A	1065	9	219.09	16.24	48	65.17	46.34	2.98
141	1032	9	168.55	6.65	48	65.17	45.81	2.10
142	1073	6	221.52	9.96	30	43.77	23.11	0.42
143	1066	9	270.47	10.14	48	65.17	44.47	1.33
144	1070	9	409.77	11.34	48	65.17	52.11	1.66

indicate the number of contacts between mineral grains. Thus the number 87 means that in this sequence, moving from left to right, there were 87 grains of microcline each of which was followed by another microcline grain in direct contact with it. The number 68 in the same matrix means that an albite grain in the sequence was followed by a quartz grain in 68 cases. The matrix with these frequencies has dimension 3x3. It can also be written as N or $N_{i;j}$. In the notation $N_{i;j}$, i denotes the mineral from which the transition to a grain of mineral j has been made.

The next matrix in the table for the same specimen is $N_{i,j;k}$. This matrix has 9 rows and three columns with transitional frequencies. Each row has its own index, one for every possible combination of two minerals (states). A row in the matrix $N_{i,j;k}$ gives the number of times in the sequence that two adjoining grains of minerals i and j were followed by a third grain of mineral k which was in direct contact with the grain of j. The composition of this third grain (k) is indicated at the top of the corresponding column. For example, 11 on the left side of $N_{i,j;k}$ indicates that this particular row contains the results of counting the number of cases that the grains Or, Q or Ab had immediately to the left of them two grains of microcline (orthoclase). There were 24 cases that the combination Or Or was followed by another Or. Similarly, there were 43 transitions from Or Or to a quartz grain (Q or 2). The other numbers in this matrix can be interpreted in the same way. The number 2 at the intersection of the row 33 with column 3 indicates that in the entire sequence there are two places where three grains of albite were adjoining each other with direct contacts between them on the straight line used for counting transitions.

Appendix 3 presents tables which correspond to the matrices of transitional frequencies $N_{i,j,k;\ell}$. These matrices correspond to the situation where knowing the compositions of three grains in direct contact with one another the transition of this triplet to a fourth grain is defined. Thus for sample 25 in Table 2B the row labelled 321 indicates the transitions from a triplet in which the first grain is albite, the second grain quartz, and the third grain microcline; intersecting row 321 is 7 under column 1 and 29 under column 2. This indicates that from 36 triplets starting with 321, 7 were transitions to microcline grains and 29 to quartz.

Below the matrix $N_{i,j,k;\ell}$ for sample 25 (Table 2B), there are two rows. The first row is labelled f_p indicating how many grains of each mineral there are in the sequence if we count the number of grains from the matrix $N_{i;j}$. For example, 348 means that the total for the second row of the matrix $N_{i;j}$ for sample 25 is equal to 348 (348 grains of quartz).

Row f_t, which occurs below row f_p, contains the numbers of grains of each mineral for a composite sequence for the specimen without consideration of the interruptions marked by semicolons. If we write the number of grains of a specific mineral h in row f_p as $n_p^{(h)}$ and in row f_t as $n_t^{(h)}$ it is obvious that $n_p^{(h)} < n_t^{(h)}$. This merely reflects that the sequence is composed of subsequences.

The material in Appendixes 2 and 3 permits us to check hypotheses about the properties of the investigated random sequences and to check their agreement with the model of ideal granite. This has been accomplished by using the procedures described in the previous section all of which have been incorporated in the program in Appendix 1. The results are shown in Tables 3, 4, and 5 all of which have similar headings. The first part, A, of each table is for specimens taken from the stock, and the second part, B, is for specimens from the dykes. The headings of the columns have the following meaning. The sample number is given in the first column; the length of the sequence, that is, the total

Table 4. Sequences of grains of orthoclase, quartz and plagioclase similar to first-order Markov chains (non-ideal granites). A-stock; B-dykes

				Λ for Markov chain order alternatives		χ^2-test for homogeneity			χ^2-test for ideal granite matrix
Specimen number	Length of sequence (in number of grains)	Number of subsequences	H_0:H_1 $\chi^2_{0.05} = 9.49$	H_1:H_2 $\chi^2_{0.05} = 21.03$	Number of degrees of freedom	Theoretical $\chi^2_{0.05}$	Observed χ^2	$\chi^2_{0.05} = 3.84$
A								
3	1109	14	312.46	11.99	78	99.62	85.05	4.59
17	1048	18	106.06	6.54	102	126.57	122.19	4.76
36	1137	16	358.83	17.17	90	113.14	93.23	5.56
44	1096	9	148.64	8.93	48	65.17	29.92	5.19
45	1061	8	145.93	9.63	42	58.12	38.13	7.84
52	1113	9	117.11	12.85	48	65.17	40.89	4.08
96	1110	18	130.91	12.62	102	126.57	114.44	5.84
112	1134	7	254.18	8.18	36	51.00	61.65	6.62
B								
31	110i	14	338.30	11.32	78	99.62	78.92	9.33
140	1058	9	1389.20	16.09	48	65.17	58.67	6.42
147	1086	11	249.24	11.84	60	79.08	98.22	7.35

Table 5. Sequences of grains of orthoclase, quartz and plagioclase similar to second-order Markov chains. A-stock; B-dykes

	Specimen number	Length of sequence (in number of grains)	Number of subsequences	Λ for Markov chain order alternatives			Λ for alternatives for partial transitions $\chi^2_{0.05}=9.49$			χ^2 - test for homogeneity	
				$H_0:H_1$ $\chi^2_{0.05}=9.49$	$H_1:H_2$ $\chi^2_{0.05}=21.03$	$H_2:H_3$ $\chi^2_{0.05}=51.00$	Or	Q	Pl	$\chi^2_{0.05}$	χ^2
A	1	1109	18	359.71	22.37	21.57	15.25	7.03	0.04	347.80	181.60
	10	1083	9	107.24	23.76	27.85	3.24	13.96	6.56	173.00	129.42
	11	1058	12	251.67	38.18	36.78	8.53	13.67	13.99	231.80	214.67
	25	1062	16	326.53	26.51	42.44	8.67	9.83	8.01	309.30	261.60
	28	1147	6	156.17	24.11	38.48	0.31	20.76	3.04	113.14	88.99
	29	1109	11	149.65	31.27	27.84	17.58	10.85	2.84	212.30	178.28
	30	1131	12	165.79	23.92	33.54	10.27	9.61	4.04	231.80	194.84
	37	1099	9	227.62	28.28	41.24	16.62	5.38	6.27	173.00	149.65
	47	1105	11	101.15	40.11	38.11	2.80	32.31	5.00	212.30	173.09
	49	1095	11	187.49	27.57	33.39	12.99	11.60	2.98	212.30	144.03
	58	1081	11	135.50	26.28	23.09	7.69	11.31	7.07	212.30	144.03
	73	1062	4	92.37	23.86	27.03	7.47	8.27	8.11	72.15	66.98
	87	1099	8	165.67	30.27	34.96	0.70	24.73	4.83	153.20	102.51
	115	1016	20	233.14	29.83	29.95	8.36	12.12	2.35	386.10	287.44
B	132	1103	10	159.58	24.52	49.30	6.64	12.01	5.86	192.17	161.71
	121	1100	10	207.05	26.53	48.50	5.85	16.39	4.26	192.17	139.30
	123	1098	10	151.87	22.79	35.12	2.77	15.29	4.73	192.17	122.24
	141A	1086	11	311.25	21.26	27.66	3.91	9.94	7.41	212.30	111.88
	145	1107	13	164.74	30.06	35.94	4.74	5.19	20.12	251.30	119.07
	146	1112	11	127.31	24.04	29.91	8.49	10.51	5.04	212.30	111.15

number of grains is given in the second column; for example in Table 3, second column, the number 1125 indicates that 1125 transitions were in the sequence for sample 2. The third column indicates how many subsequences occur in the composite sequence; for example, the number 15 in the third column of Table 3 indicates that the sequence was composed of 15 subsequences. The next columns contain the likelihood ratio Λ for the different Markov hypotheses. Values of Λ are given for the alternatives $H_0:H_1$ and $H_1:H_2$ in Tables 3 and 4, and $H_0:H_1$, $H_1:H_2$, and $H_2:H_3$ in Table 5. Under the heading $H_0:H_1$ ($\chi^2_{0.05} = 9.49$) if $\chi^2 < 9.49$, H_0 can be accepted with probability of an error of the first kind equal to 5 per cent. If $\chi^2 > 9.49$ we accept hypothesis H_1 instead of H_0. In the first row of Table 3 the value of χ^2 in this column is equal to 180.38 and we accept H_1 as the alternative. The other headings have similar meanings. If the sequence is homogeneous (see later), the data in the columns for $H_i:H_{i+1}$ ($i = 0, 1, 2$) permit us to draw conclusions about the Markov order of the sequence in a statistical sense.

The next three columns indicate whether the sequence is nonhomogeneous or homogeneous. The figures shown were calculated for the specific case of the particular Markov hypothesis that was accepted. At the beginning we have information on the numbers of the degrees of freedom, which are different for different specimens, as even if the sequences have the same Markov order, they are composed of different numbers of subsequences. It is obvious also that the number of degrees of freedom is different for Markov sequences of different orders. There follows the theoretical value of χ^2 for the 5 per cent level of significance for the number of the degrees of freedom. The observed value of χ^2 is given in the next column. We can accept that the sequence is homogeneous if the theoretical value of χ^2 is higher than the observed value of χ^2. The testing of the hypothesis of homogeneity was performed by using the conventional χ^2 method as explained in the previous section.

The last column of Tables 3 and 4 contains the observed value of χ^2 obtained by comparison of the observed matrix $P_{i;j}$ with the matrix $C\Lambda_p$ which is composed of the calculated values of c_{ij}. This is the result of using the method of minimum χ^2 for the condition that $c_{ij} = c_{ji} > 0$. The matrix

$$\Lambda_p = \begin{bmatrix} P(a_1) & 0 & 0 \\ 0 & P(a_2) & 0 \\ 0 & 0 & P(a_3) \end{bmatrix}$$

was introduced for obtaining the values used for comparison with the matrix $P_{i;j}$ whose elements are observed values of $P(a_i \mid a_j)$; $i,j \in \{Or, Q, Ab\}$. These values were also obtained from the matrices $N_{i;j}$

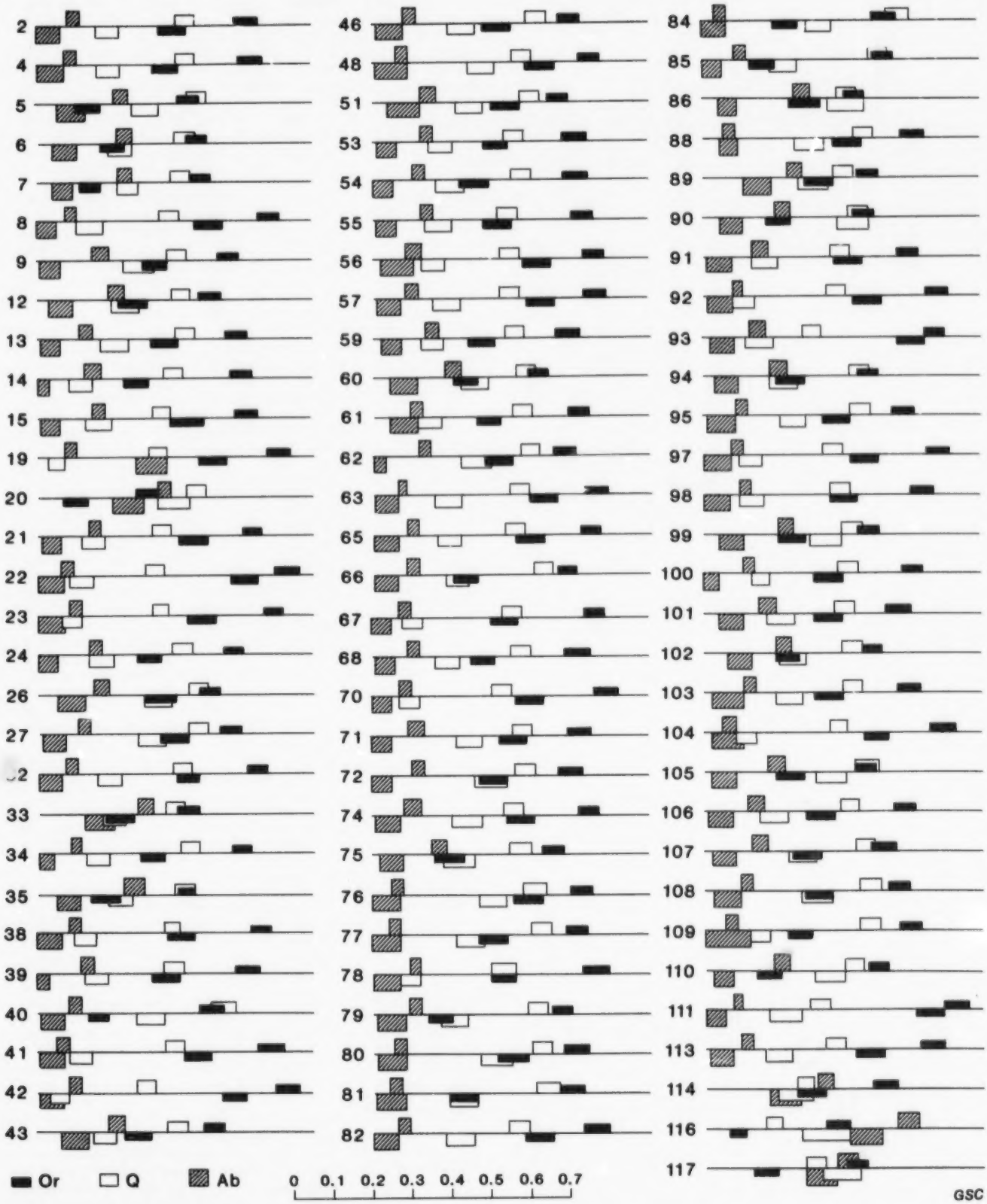

Figure 9. *Confidence intervals (α = 0.05) for probabilities P_i (above straight lines) and P_{ii} (below straight lines) for microcline, quartz and albite in the Meech Lake aplite stock.*

listed in Appendix 3. If $\chi^2 < 3.84$ for the comparison of $P_{i;j}$ and $C\Lambda_p$ we suppose that the investigated sequence can indeed be a sequence of grains from an ideal granite, as explained in the previous section.

Tables 3 to 5 provide the main test values which should be used for the investigation of igneous granites when it is required to answer the question: are there traces of secondary reworking of primary granite with the generation of new grains of microcline, quartz, or albite, whilst the primary texture of the magmatic granite was retained? Finally, it is necessary to check the interrelations between $P(a_i \mid a_i)$ and $P(a_i)$. This test has been carried out on the basis of equation (17) by means of a diagram first used by Romanova (1976) in her investigation of granitic rocks in Newfoundland. Diagrams of this type are given in Figures 9 and 10. This graphical method can be explained as follows, taking Figure 9 for example. At the bottom of Figure 9 there is a scale according to which $P(a_i)$ and $P(a_i \mid a_i)$ are expressed along straight lines numbered on the left side of the diagram to indicate the specimens plotted in Figure 1. Along each straight line are two groups of rectangular boxes. Those above the straight line are intervals calculated from equation (15) for $P(a_i)$ of microcline (black), quartz (white), and albite (hatched). Below the line the same intervals are plotted for $P(a_i \mid a_i)$ using the same patterns. If for a given specimen all three rectangular boxes above the straight line are located to the right of the corresponding boxes below the straight line, the model of ideal granite, according to this test, is consistent with the observations. A more detailed discussion of Figures 9 and 10 is given in the next section.

The Meech Lake Aplites and the Ideal Granite Model

To evaluate the hypothesis that the Meech Lake aplites are magmatic rocks and were generated by direct crystallization from a magmatic melt, we can test the observations against the statistical model developed for ideal granite. The purpose of these tests is also to check whether the aplites remained unchanged after crystallization, or whether they were metasomatically altered with the generation of new grains of microcline, albite, and quartz. Table 3 contains the results of checking the statistical hypotheses for the type of random sequence of grains of Or, Q, and Ab when this sequence can be distinguished from a simple Markov chain. The observed matrix $P_{i;j}$ statistically coincides with the matrix $C\Lambda_p$ where C is a matrix of coefficients c_{ij} and Λ_p is a diagonal matrix with the elements $P(a_i)$ on its main diagonal equal to the elements of the vector of stationary probabilities. A study of the data of Table 3 indicates that the Bernoullian hypothesis of the alternative H_0:H_1 is rejected in all cases with a very high level of significance, although Λ only rarely assumes values which are typical of Precambrian granites (Vistelius et al., 1977; Podkovyrov, 1978). The values for H_0:H_1 in Table 3 are typical of fresh high-level magmatic granites (Romanova, 1978; Ivanov, 1978), but they are higher than those of

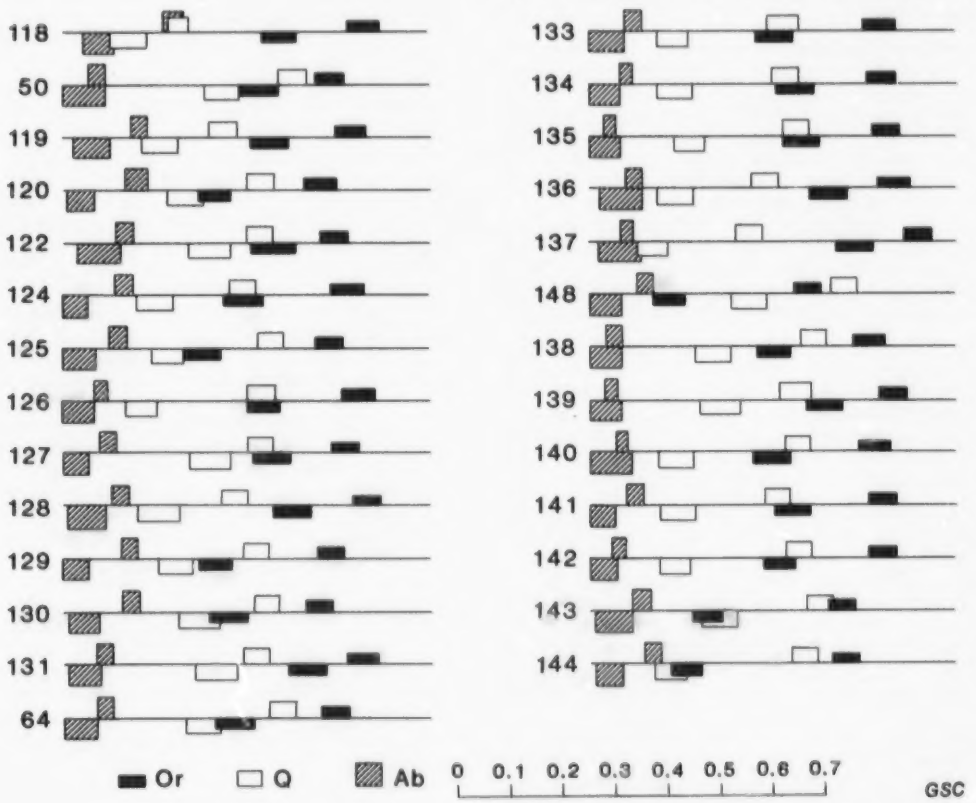

Figure 10. *Confidence intervals ($\alpha = 0.05$) for probabilities P_i (above straight lines) and P_{ii} (below straight lines) for microcline, quartz and albite in aplite dykes in Meech Lake area.*

granites enriched with volatile components (Vistelius and Romanova, 1976). These values, however, differ greatly from those of degenerated ideal granites. The next alternative H_1:H_2 is usually resolved in favour of H_1. Specimens with values of Λ for H_1:H_2 which are contained between 0 and 21.03 are shown in Table 3. In general, the smaller a value of Λ for this alternative, the more strongly the hypothesis H_2 of a second order Markov chain is rejected. Typical values of Λ for H_1:H_2 in Table 3 are about 10 or 15. In the stock (Table 3A) there are three specimens (samples 33, 34, 76) with $\Lambda > 20$ and in dykes (Table 3B) there are two (samples 136 and 139). The results of these computations indicate that 96 specimens of the stock rocks and 30 specimens of the dykes are similar to simple Markov chains. In other words, the first condition for the model of ideal granite is fulfilled.

The last column in Table 3 gives the values of χ^2 for the comparison of $P_{i;j}$ with $C\Lambda_p$. If the conditions (equations 8a-e) of the model are fulfilled, the estimated value of χ^2 should be smaller than the value 3.84 given at the top of the column. It is clear from Table 3 that this condition of the model is fulfilled for all specimens from the stock. Three values (samples 50, 86, 99) are greater than 3.0. For the dykes also three specimens (samples 118, 120, 133) have χ^2 values which are somewhat higher than 3.0. Since these values are sampling values resulting from random variables, the goodness of fit of approximating $P_{i;j}$ by $C\Lambda_p$ should be accepted as sufficient. In other words, the second condition of the ideal granite model is fulfilled for all specimens listed in Table 3.

Table 3 also contains observed values of χ^2 for checking the hypothesis that a sequence is a homogeneous simple Markov chain against the hypothesis that it is a nonhomogeneous simple Markov chain. Comparison between the column for $\chi^2_{5.05}$ (table values for 5 per cent level of significance) and that for observed values of χ^2 shows that most of the sequences are homogeneous simple Markov chains. Only 11 out of all 88 specimens of the stock rocks have a level of significance for acceptance of the homogeneity hypothesis which is less than 5 per cent. This level of significance is lowest for sample 103 which has about 0.005; in all other cases it is much higher. For the dykes only sample 136 has a level of significance below 0.05 ($\alpha \approx 0.01$).

At the beginning of this paper it was pointed out that there are indications that the stock rocks have a heterogeneous composition and this could have been reflected in our estimators of homogeneity. However, as we have seen, on a local scale the sequences of grains of the investigated specimens are sufficiently homogeneous. Thus the third condition for the model of ideal granite is fulfilled.

The last condition of the model is related to the special property of the simple Markov chains of an ideal granite whose Markovity is generated by the inequality $P(a_i \mid a_j) < P(a_i)$. If this inequality is realized for all values of i (=1,2 or 3) we have the special Markov chain corresponding to the ideal granite model. Figure 9 for the stock rocks, and Figure 10 for the dykes show that the inequality is very well satisfied for most of the specimens. There are only two cases (samples 116 and 117) where $P(a_i \mid a_j)$ for quartz (i=2) is higher than $P(a_j)$. In the worst possible situation, not encountered here, the inequality is satisfied for one mineral (usually Or) only. In general, other studies have shown the inequality is most stable for quartz which usually satisfies the condition whereas albite violates the inequality more frequently than the other two minerals.

The preceding discussion of Table 3 and Figures 9 and 10 has shown that most of Meech Lake aplite can be regarded as a typical ideal granite. There are about 10 specimens from the stock with $P_i \approx P_{ii}$ for albite, and 5 similar cases for quartz. These can be neglected since in these cases $P_i \approx P_{ii}$ does not reject $P_i < P_{ii}$. In other words, the Meech Lake aplites possess the typical features of fresh, unaltered high-level granites and are directly comparable to the Kyzyl-tas granites in Kazakhstan (Ivanov, 1978), or the Marlsburg granite in the Federal Republic of Germany.

Nonideal Granites with Markov Chains of the First Order

Only the fulfillment of all requirements of the model of ideal granite permits us to classify a specimen as an ideal granite without secondary grains of Or, Q or Ab. In Table 4 the results of checking the ideal granite hypotheses are shown for the case where the sequence is a simple Markov chain but the comparison of $P_{i;j}$ and $C\Lambda_p$ shows that the matrix $C\Lambda_p$ does not fit $P_{i;j}$ if the matrix C is made to satisfy the conditions prescribed by the model. The values of χ^2 in the last two columns indicate that these specimens are not ideal granites although the other properties of ideal granite are satisfied. The Bernoullian hypothesis is rejected with high levels of significance and the values of Λ for the alternative H_1:H_2 are typical for the case that H_2 should be rejected. These sequences are practically homogeneous. We will refer to the rocks listed in Table 4 as rocks with traces of alteration.

The second group of rocks, which are different from ideal granite, is listed in Table 5. These rocks have sequences of grains similar to second order Markov chains. The sequences are homogeneous. When we study the values of Λ for the alternative H_1:H_2, it is clear that we should reject the hypothesis H_1 although most values are relatively small. Only samples 11, 29, 47, and 87 have Λ greater than 30 which permits the rejection of the hypothesis H_1 without much doubt. In the other cases, the result is somewhat uncertain.

For clarification of the problem of metasomatic alteration of the rocks we have plotted the data from Tables 4 and 5 on the map of the Meech Lake aplite stock in Figure 11. It could be that the metasomatically altered rocks (nonideal granites) are located within relatively narrow zones with directions of strike that are perpendicular to one another (NNE-SSW and WNW-ESE). It is possible that the secondary alteration reflects postmagmatic activity. The intensity of the metasomatic alteration can be estimated from the data listed in Table 5 under Λ for alternatives for partial transitions. These data indicate that the simple Markov transition was lost either for one mineral or for two minerals. Because the alteration was weak, as shown by the small values of Λ, and because the area underlain by altered rocks is small, this case was not investigated in detail. Special investigations of situations where metasomatic alteration of granitic rocks is important can be found in the study of Arga-Ynykh-Khaya rocks in Yakutya by Romanova (1978).

Alteration of Aplitic Rocks and Carbonatites

It has been mentioned that the investigation of tin-bearing granitic massifs which contain high-temperature hydrothermal mineral deposits such as Carmenellis, Iul'tin, Arga-Ynykh-Khaya, and Omsurchan has shown that these massifs have a Markov structure of the grain sequences that differs from that of barren massifs without such deposits (Marlsburg granite, Kyzyl-tas massif in Kazakhstan, and also the Chalba intrusion near the town of Amursk in the Khabarovsk region). The metasomatic alteration accompanying the genesis of high-temperature hydrothermal deposits influences the values of Λ in the alternative H_1:H_2 for H_2, or the values of Λ in H_2:H_3 for H_3. It is interesting to determine whether or not the carbonatites and pegmatites within the Meech Lake aplitic stock influenced the sequences

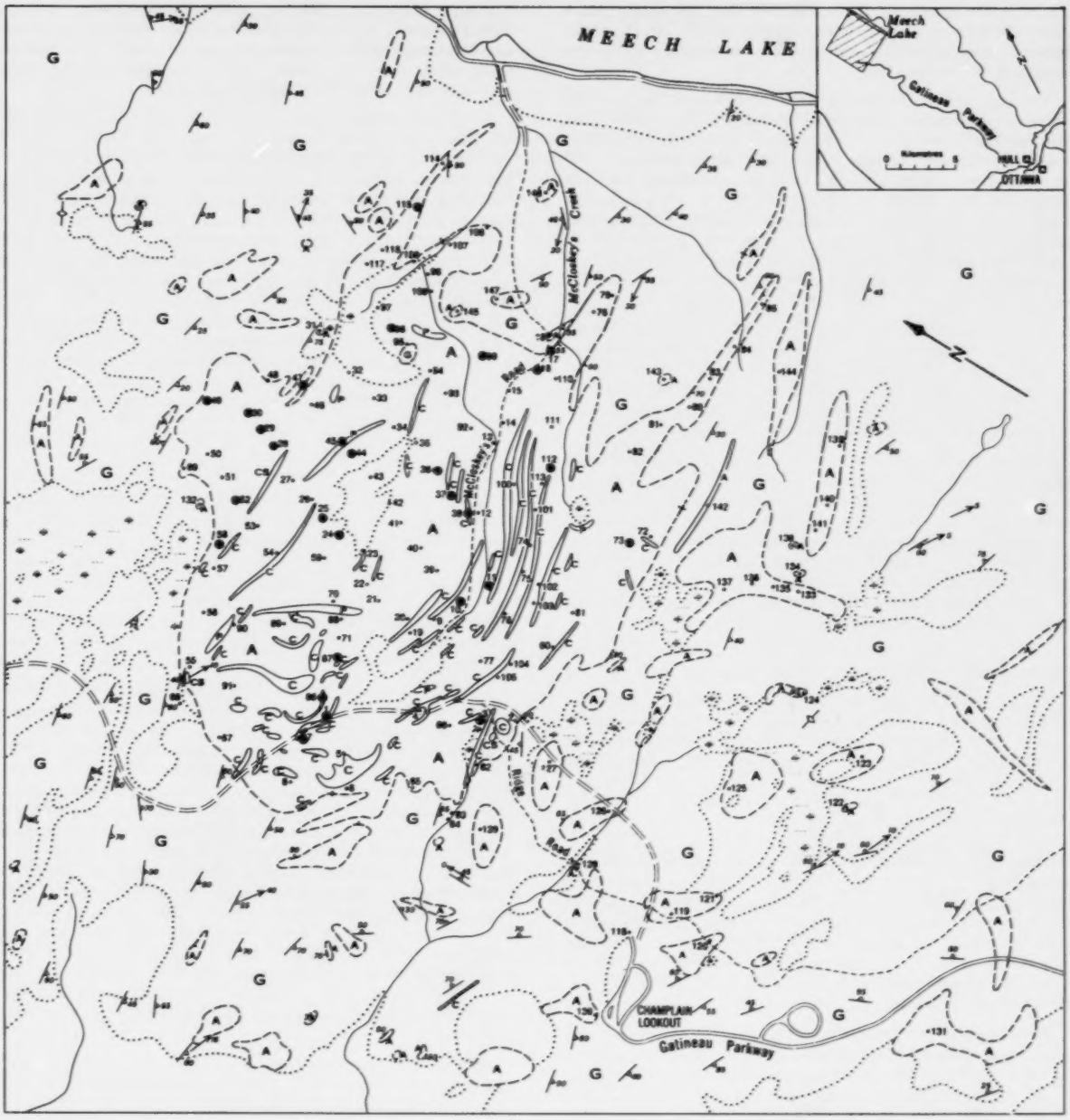

Figure 11. *Locations of nonideal granite samples in Meech Lake aplite stock.*

of grains in the nearby aplite. Suppose that the latter has a primary Markov structure with, superimposed on it, some secondary mineralization which is connected with the carbonatites or pegmatites.

We can see in Figure 11 that samples 7, 38, and 86 occur near carbonatites whereas samples 24, 29, 30, 49, and 99 are farther from them. Sample 45 is near a pegmatite body whereas sample 52 was taken far from pegmatite. All these specimens have similar Markov properties for the sequences of the grains of Or, Q, and Ab. Their values of Λ for the alternative $H_1{:}H_2$ are about the same. Consequently, study of Figure 11 does not permit us to conclude that secondary grains of quartz, microcline or albite were generated at the same time as the carbonatites. As far as we can see, the same holds true for the pegmatites.

Because carbonatites do not occur in the aplitic dykes, it is possible to check for a correlation between specimens originating from the stock and those from the dykes by comparing the type of the sequence of grains of Or, Q and Ab in them (ideal granite, non-ideal granite with simple Markov chain, or non-ideal granite with second order Markov chain).

Table 6 shows the distribution of our observations as a contingency table. Computations show that the χ^2 value for Table 6 is equal to 0.31. Consequently there is no correlation between type of sequence of grains and presence of carbonatites in the lithological unit that was sampled. If our observations are independent, the lack of significant correlation should be accepted as an argument in favour of the absence of secondary alteration of the aplites under the influence of the carbonatites.

Geological Interpretation of the Parameters c_{ij}

In equation (8e) we postulated an important feature of crystallization in a closed system, i.e. one without supply of additional material from outside the magma chamber. We accepted that in the closed system the transitional probability satisfies

$$P(a_i^h \mid a_j^{(h+1)}) = c_{ij}\, P(a_i^{(h)})$$

where the c_{ij} are constants with properties indicated by equation (8e). Thus c_{ij} should reflect the main characteristics of the process of crystal growth in the aplite. It seems natural to believe that in a dry viscous magma it is difficult to supply material for crystal growth. In this case c_{ij} will differ from 1. If the magma is enriched by volatile components, which facilitate the process of diffusion, c_{ij} should be equal, or close, to 1.

Investigation of the rocks of a granitic body in Arga-Ynykh-Khaya showed that, in the part of the body where there were abundant volatile components in the magma, the value of c_{ij} satisfies $c_{ij} \approx 1$ for all i and j (Vistelius and Romanova, 1976; Romanova, 1978). These rocks were referred to as degenerated ideal granites by Vistelius and Romanova (1976). Similar interrelations were found by Vistelius and Choubert in the Marlsburg granite, by Vistelius and Harbaugh (1980) in the granites of the Yosemite Valley, California, and by Ivanov and Vysotzky in the Chalba intrusion near Amursk, USSR.

Thus the parameter c_{ij} may reflect the characteristics of the process of crystallization. If the CO_2 of the carbonatites of the Meech Lake aplite stock was dissolved in the magma before its crystallization, it is to be expected that this would be reflected by the c_{ij} values of the rocks. As was pointed out before, most of the rocks of the Meech Lake aplite stock are ideal granites. It is possible in different ways to characterize the behaviour of all values of c_{ij} simultaneously in a specimen. For example, the value of Λ computed for the alternative $H_0{:}H_1$ provides a measure of deviation of the specimen with c_{ij} from the Bernoullian case. Suppose that c_{ij} denotes the set of random variables of which the c_{ij} values of a specimen are a realization. It is then possible to show that in the Bernouillian case the sample values of c_{ij} for the specimen have an expected value equal to 1, and variance

$$\sigma^2(c_{ij}) = N \sum_{i,j} P_{ij}\,(c_{ij}-1)^2$$

If we want to characterize the process of crystallization of the rock-forming minerals independently of their composition we can make use of the measure

$$\delta_c = \Sigma\,(c_{ij} - 1)^2$$

Table 7 contains the values of δ_c for the ideal granites of Meech Lake. All individual values of c_{ij} that were used are listed in Appendix 4.

The average values of δ_c of granites from different regions of the world are given in the Table 8 for comparison. The table contains δ_c values for the degenerated ideal granites of Arga-Ynykh-Khaya (rows 1-3). Rows 4 to 9 are for rocks of the composite dyke at Tinkan Bay in the Sea of Japan. In this dyke two different rocks occur in the same fissure. One rock is a fine grained granite with biotite. The second rock is aplite, with a granophyric texture locally; it is exceedingly fine grained and without dark minerals. The content of volatiles in the granite (67 specimens) was probably much higher than that in the aplites. Rows 10 to 17 present some new data on δ_c for granitic rocks of Precambrian age. If these relatively few data are indeed representative it seems that in the Precambrian most granites were generated from dry and viscous magma as indicated by the high values of δ_c. If we compare data for δ_c of the Meech Lake aplites with δ_c for rocks from Table 8, we see:

(a) The degenerated ideal granites, which, as we assume, were generated from magma enriched in volatile components, have δ_c values of less than 1.67. These are of the same order of magnitude as the values of Meech Lake aplites (dark pattern in Fig. 12). These values are much smaller than those given in Table 8 for the aplites from the composite dyke near Tinkan Bay, and they are somewhat higher than those of the granitic rocks at Tinkan Bay.

(b) If we assume that the Tinkan composite dyke rocks are representative of granites generated from magma enriched in volatile components, whereas the aplites were generated from dry magma, we should also

Table 6. Contingency table for type of sequence of grains and form of aplite body

Form of aplite body \ Type of sequence of grains	Ideal granite	Non-ideal granite with simple Markov chain	Non-ideal granite with second order Markov chain	Sum
Stock	88	8	14	110
Dykes	27	3	6	36
Sum	115	11	20	146

Table 7. δ_C values for Gatineau aplites similar to ideal granites. A-stock; B-dykes

Specimen number	χ^2 for comparison of $P_{i;j}$ with cAp ($\chi^2_{0.05} = 3.84$)	δ_C
A		
2	0.0584	1.6479
4	1.0088	2.5082
5	0.1814	1.6469
6	0.9499	1.4770
7	2.7191	2.2416
8	0.9825	1.6444
9	0.2877	1.6811
12	0.6504	1.3148
13	0.1750	1.4634
14	1.2825	3.0572
15	0.8658	1.3454
19	2.0897	2.4760
20	1.9297	0.7864
21	2.2879	1.5228
22	0.4503	1.1420
23	0.3358	2.9266
24	2.5625	2.0325
26	2.5608	0.5547
27	0.0234	0.9272
32	1.9819	1.9048
33	0.3836	0.9716
34	0.0948	2.1515
35	0.9682	1.4207
38	0.1387	2.4318
39	0.5487	2.2656
40	0.6306	2.4600
41	0.4060	2.5295
42	0.4818	2.3351
43	0.0349	1.1785
46	1.5210	1.2099
48	1.4486	0.9808
50	3.3492	0.6567
51	1.7375	0.6497
53	0.3628	1.7761
54	1.6087	3.2963
55	0.2200	2.1575
56	0.5241	0.9135
57	1.6896	1.1987
59	0.2621	1.7679
60	0.0736	1.1509
61	1.4949	2.4501
62	0.0794	1.6192
65	1.4270	1.7536
66	1.2712	2.3183
67	0.1750	3.1193
68	0.3560	2.8848
70	2.5778	3.0681
71	0.0268	1.7896
72	1.9459	2.8006
74	0.7374	2.3282
75	0.0166	2.2339
76	0.0988	1.4932
77	1.0264	2.9628
78	0.1852	3.6003
79	0.0347	3.2136
80	0.4732	2.6802
81	0.1295	4.6592
82	0.3289	1.6397
84	0.0755	2.5306
A		
85	2.0832	2.8169
86	3.0462	1.1406
88	2.0352	2.9269
89	1.5103	0.4890
90	0.0056	1.9490
91	0.7121	1.3016
92	1.2410	2.3741
93	0.5309	0.8518
94	0.0386	1.4317
95	0.2146	1.1369
97	0.5083	2.5314
98	0.0981	2.3630
99	3.4676	1.3456
100	2.4096	2.0964
101	2.3111	1.1464
102	1.1128	1.2851
103	0.1410	1.7164
104	0.8065	2.2704
105	0.5467	1.4307
106	0.0927	1.7812
107	0.8766	1.2962
108	2.2076	1.8026
110	1.7817	2.8160
111	1.3430	0.9803
113	0.6337	2.4002
114	0.4326	0.9304
116	0.2201	1.8598
117	0.9538	1.3609
B		
63	0.1446	1.4731
64	0.7233	2.9269
109	0.2960	3.1521
118	0.3261	1.6632
119	1.3544	1.3871
120	3.1652	1.7859
122	0.7710	0.6389
124	0.1668	2.4619
125	0.0495	2.5111
126	0.1266	2.1490
127	1.8568	1.9810
128	0.6453	1.4734
129	0.3356	2.3377
130	0.1411	1.3042
131	0.0729	1.8522
133	3.4844	1.8230
134	0.5801	1.6690
135	0.0744	2.1094
136	0.1680	0.8599
137	1.7870	2.1754
137A	1.0702	1.9860
138	0.5321	2.3376
139	0.5184	2.5053
140A	2.9769	1.5665
141	2.0964	1.9964
142	0.4163	2.2579
143	1.3322	2.7720
144	1.6632	3.0074

Table 8. δ_C values for other granitic rocks

	Region and age	Type of rock	Number of specimens	χ^2 for comparison of $P_{i;j}$ with $C\Lambda p$ ($\chi^2_{0.05} = 3.84$)	δ_C
1	Arga-Ynykh-Khaya, Yakutiya; Cretaceous	Leucocratic, fine grained granites (Romanova, 1978)	34	2.31	0.04
2			72	0.28	1.75
3			97	0.16	0.06
4	Tinkan Bay Japan Sea; Upper Paleozoic	Granite with biotite	67	0.31	0.12
5		Aplite	60	3.23	1.94
6		Other rocks from	66	3.16	2.14
7		Composite dyke	69	0.62	2.62
8		(Vistelius and	71	0.50	2.11
9		Romanova, 1979)	71	1.18	1.52
10	Yllymakh River, Aldan shield; Lower Precambrian	Coarse grained granites (Vistelius, et al., 1977)	2	1.99	3.92
11			3	0.34	4.12
12			6	0.15	3.22
13			8	0.86	3.87
14	Vyborg Rapakivi massiv; Upper Proterozoic	Matrix of Rapakivi granites (Demina, 1974; Podkovyrov, 1978)	1	0.73	3.89
15			2	0.66	4.36
16			3	0.33	4.01
17			8	0.44	4.44

assume that the smallest values of δ_C in the Meech Lake aplite belong to rocks relatively enriched in volatile components.

(c) The values of δ_C for the Meech Lake rocks, in comparison with typical values of δ_C for Precambrian granites, have maximum values of the same order of magnitude as Precambrian granites and somewhat higher than the aplites of Tinkan Bay. Therefore, it seems that part of the Meech Lake aplite was generated from a dry, viscous magma.

Figure 12 shows a subdivision of the aplites within the stock corresponding to their δ_C values. It seems that the smallest values of δ_C occur in the northern part of the stock whereas most of the high values of δ_C are located in the southern part of the stock and in its eastern apophyses.

If we assume that low values of δ_C are indicators of enrichment of magma by volatile components and high δ_C values are indicators of dry magma we are led to believe that there was an uneven distribution of volatile components in the stock.

GENERAL CONCLUSIONS

Observations at the contacts of the investigated stock with country rocks indicate that the stock intruded the country rocks. There are possible blocks of country rocks within the stock which may be large zenoliths. Contacts with the surrounding orthogneiss are sharp and the aplite clearly transects the foliation of the orthogneiss. Numerous aplitic dykes with compositions similar to that of the stock rocks occur in the country rocks around the stock. We assume that these direct geological observations show that the stock rocks were generated from a melt with crystallization taking place in a magma chamber within the surrounding orthogneiss where the stock is located at present.

The model of ideal granite was found to be applicable to most of the stock rocks. Thus we believe that this main part of the stock is represented by unaltered magmatic aplite. Locally a minor secondary development of quartz, microcline, and albite occurred; this is inferred from grain sequences that indicate second order Markov chains.

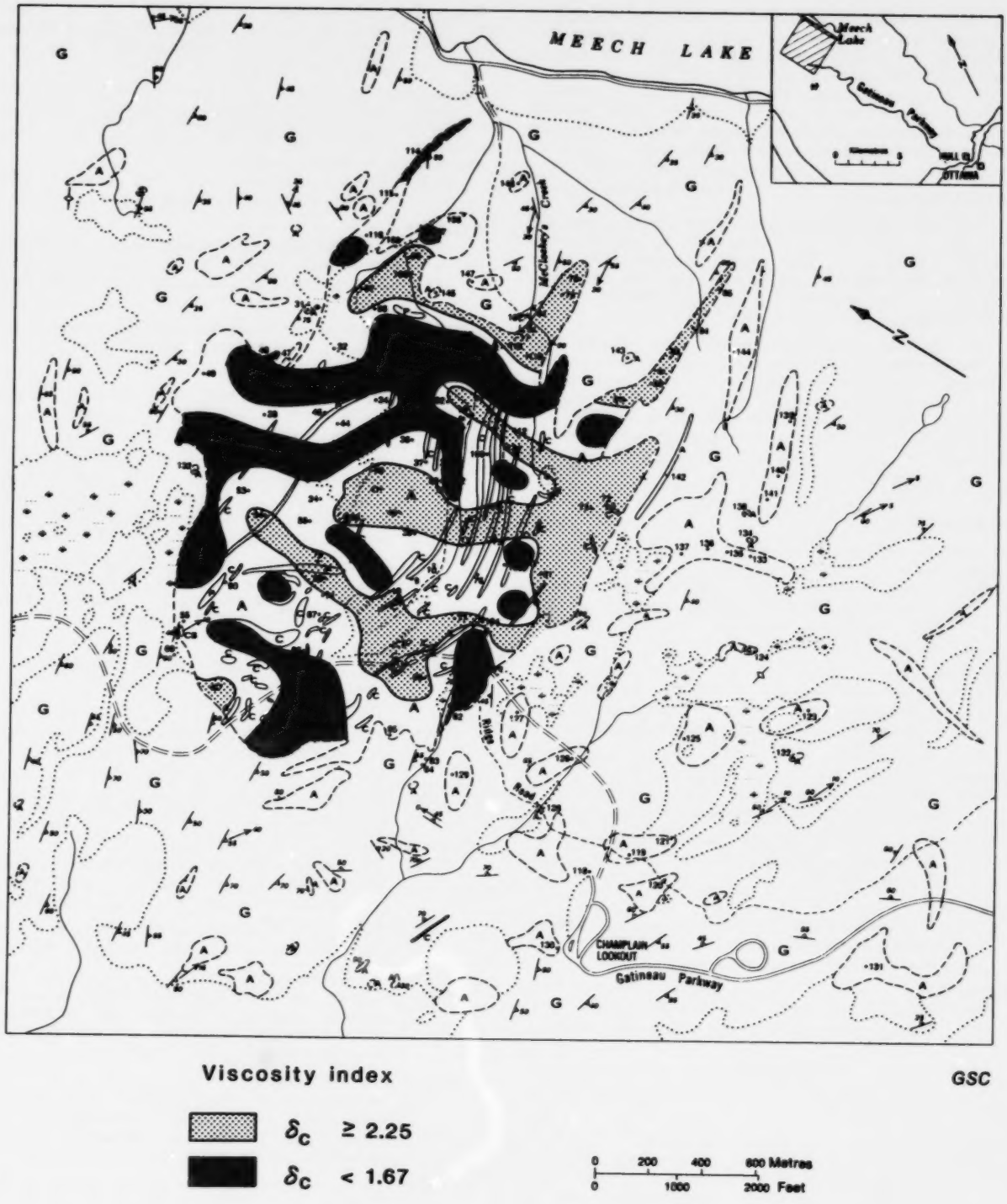

Figure 12. *Contour map of the viscosity index δ_c. Contour values of 1.67 and 2.25 were selected because they represent approximately 33% and 67% fractiles of the frequency distribution of δ_c. Lowest values (<1.67) indicate area where magma was less viscous; highest (>2.25) indicate magma of higher viscosity.*

The parameters c_{ij} of the model reflect the viscosity of the magma and its content of volatile components. The distribution of volatile components in the stock magma before its solidification was not uniform. The northern part of the magma chamber became enriched in volatile components in comparison with the southern part. Neither the carbonatites nor the pegmatites caused secondary metasomatic transformations of the aplite stock.

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A.G. Fabbri, Geological Survey of Canada, and R. Kretz, University of Ottawa, participated in the sampling of the aplitic rocks in the field. A.G. Fabbri in particular collected 36 specimens used for this study. The computer program in Appendix 1 was converted from ALGOL into FORTRAN by K.F. Chong, Computer Science Centre, Department of Energy, Mines and Resources, Ottawa, and S.N. Lew, Geological Survey of Canada.

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APPENDIX I

FORTRAN IV computer program MARKOV

Users' Guide to the program MARKOV

For the purposes of this guide, it is assumed that punch-cards are used to enter the program and data. The procedure is similar for users of computer terminals who put programs and data on disc files.

The FORTRAN IV program, MARKOV, which is listed at the end of this appendix consists of 1232 cards. In addition to the main program, there are five subroutines: P0042, MRKOV, MRKOV2, CHISQR, and GBS002.

The first data card is reserved for a comment to be printed at the beginning of the output. All eighty columns of the card can be used for this.

Subsequent data cards must follow a more rigorous format. They contain the observed subsequences of grains. The subsequences are entered as a string of numbers 1, 2, or 3, where each number represents a different mineral. Each subsequence must be terminated by a semicolon (;) including the last subsequence. The first subsequence begins in the first column, and all eighty columns must be used on each card, except on the last card. Blanks between any of the characters are not allowed. Sample 25, which was used for an example in Table 2 of the text, is also used as the example in this appendix. If the reader wishes to use this example for testing purposes, the numbers and semicolons should be entered consecutively filling all 80 columns on the input cards, except on the last input card. There is a program-imposed limit of forty data cards, not counting the first comment card.

The control cards required are different from one installation to another. Most installations, however, require a "Job" card, an "Account" card, a card to separate the Fortran program from the data, and a card to indicate the end of the data cards. Consequently, most job decks should resemble the following:

[JOB Card]
[ACCOUNT Card]

FORTRAN
program

[a special control card]

Data cards

[a special control card]

When the program is run for the example, the output is as follows:

```
     SPECIMEN NUMBER 25

NUMBER  SUBSEQUENCE
  OF        OF
GRAINS     GRAINS
 ( 65)  321223123121313131231313131313123131312121321221222313212132121321213212
 (100)  311113232131312211212131212131311311211312231212333321312112321121111211213121212131212111221232121213
 ( 14)  121313113213211
 (110)  212131213121212132121212123111212132121213131313212121321323221313121212132311213131321213131311213112121312131
 ( 18)  121313112113123212
 (106)  111312121221312121331212133131212112122121312321313121322213121212121312121232131321221121321212121311213 2
 (110)  132122121112121223121212121212222322121312131321212131312112121212131211213112213212121131331211132121212131 2
 ( 98)  131312131313131312321321221213123121321212131232323121111212222121232121212121113123212131311112121312
 ( 18)  312131111222223131
 ( 18)  323121121121212131
 ( 84)  122312121112312213121311221221121112312212133233233121112121312132131212121321323231
 ( 80)  312121313123211312312323232121212132123312111132131213121321232112121211131321231
 ( 21)  321113211232121211213
 (107)  211231121212222113111212132331212121232323112112312121312313131212121211121131211313121231211312321211132121
 ( 25)  323211121321231213122121 2
 ( 88)  132131212123121312121211212131321213121222123132111122111231321122312312122131331131211311

NUMBER OF SUBSEQUENCES =   16

TOTAL NUMBER OF GRAINS = 1062
```

	STATE 1	STATE 2	STATE 3
NUMBER OF SUBSEQUENCES ENDING WITH	8	6	2

FREQUENCIES

FROM STATE(S)

1 GRAIN -N(I)			
F(T)=	485	354	223
F(P)=	477	348	221
2 GRAINS-N(I;J)			
1	87	245	145
2	250	39	59
3	141	68	12
3 GRAINS-N(I,J;K)			
11	24	43	20
12	177	28	35
13	100	38	5
21	45	125	78
22	23	8	8
23	31	23	5
31	17	74	44
32	48	3	16
33	7	3	2
4 GRAINS-N(I,J,K;L)			
111	4	13	7
112	30	6	7
113	14	6	0
121	32	84	60
122	18	5	5
123	19	14	2
131	11	49	36
132	31	1	5
133	4	1	0
211	15	20	10
212	93	16	13
213	52	21	3
221	5	11	7
222	3	2	3
223	7	1	0
231	4	18	7
232	14	2	7
233	3	1	1
311	4	10	3
312	52	5	15
313	33	9	2
321	7	29	11
322	2	1	0
323	5	8	3
331	1	5	1
332	1	0	2
333	0	1	1

INITIAL PROBABILITIES - IP(I)

(1)	(2)	(3)
.45668550	.33333333	.20998117

TRANSITIONAL PROBABILITIES - TP(I,J)

	(1)	(2)	(3)
(1)	.18238994	.51362683	.30398323
(2)	.71839080	.11206897	.16954023
(3)	.63800905	.30769231	.05429864

STATISTICAL SIGNIFICANCE TESTS

	0TH ORDER	1ST ORDER	2ND ORDER	3RD ORDER
REVERSIBILITY (CHI-SQUARED)		.7442	6.7365	25.5081
THEORETICAL CHI-SQUARE VALUES AT 0.05		7.8100	16.9000	51.0000
NUMBER OF DEGREES OF FREEDOM		3	9	36
MARKOVITY (CHI-SQUARE)		326.5284	26.5111	42.4362
THEORETICAL CHI-SQUARE VALUES AT 0.05		9.4900	21.0300	51.0000
NUMBER OF DEGREES OF FREEDOM		4	12	36
HOMOGENEITY (CHI-SQUARED)	10.9581	121.2100	261.4968	
THEORETICAL CHI-SQUARE VALUES AT 0.05	43.8000	113.1000	309.3282	
NUMBER OF DEGREES OF FREEDOM	30	90	270	

ALTERNATIVES FOR PARTIAL TRANSITIONS

(1)	(2)	(3)
8.6707	9.8339	8.0065

NOTE: FOR THE ABOVE, THE THEORETICAL CHI-SQUARED VALUES ARE ALL 9.49, FOR 4 DEGREES OF FREEDOM.

MATRIX OF PARAMETERS, C(I,J)

.39908696	1.54851577	1.43618046
1.54851577	.33287301	.86606640
1.43618046	.86606640	.26396849

MATRIX P(I;J) = C(I,J) * LAMBDA(P)

.18225723	.51617192	.30157085
.70718470	.11095767	.18185763
.65588279	.28868880	.05542841

IDEAL GRANITE (CHI-SQUARED) .7767
THEORETICAL CHI-SQUARED AT 0.05 IS 3.84

VERIFICATION OF SYMMETRY - FOR C(I,J)

0.0000	-.0000	-.0000
.0000	0.0000	.0000
.0000	-.0000	0.0000

VERIFICATION OF UNIT-VECTOR - C(I,I)*IP(I)
1.00000000 1.00000000 1.00000000

CONFIDENCE INTERVALS FOR IP(I) AND TP(I,I)

	LEVEL=0.05		LEVEL=0.01	
IP(I)	UPPER	LOWER	UPPER	LOWER
.4567	.4837	.4301	.4948	.4195
.3333	.3585	.3090	.3689	.2993
.2100	.2318	.1894	.2409	.1814
TP(I,I)	UPPER	LOWER	UPPER	LOWER
.1824	.2107	.1511	.2238	.1405
.1121	.1415	.0839	.1549	.0746
.0543	.0858	.0312	.0998	.0247

THE VALUE FOR DELTA-C IS 2.3660

Notes on program conversion

The FORTRAN program MARKOV is a combination of two ALGOL programs named "Verification of Markovian behaviour" and "Granite" originally published with documentation in Russian by Vistelius et al. (1972). The following three revisions were made during conversion:

1. In the original programs an array of chi-squared values from Owen (1962) is used, together with a set of linear interpolation procedures, so as to obtain all chi-squared values with the number of degrees of freedom going from 1 to 1000. In the FORTRAN program MARKOV, table values are used for the first 100 degrees of freedom and the Wilson-Hilferty approximation formula (see Johnson and Kotz, 1970, p. 176) is used for larger numbers of degrees of freedom. The program MARKOV, therefore, also provides answers for long sequences in places where the number of degrees of freedom exceeds 1000.
2. The numbers of degrees of freedom in (a) reversibility test, and (b) ideal granite test for comparison of the matrices $P_{i;j}$ and $C\Lambda_p$ used in the program MARKOV are in agreement with the results given in the text of the present paper and in other recent publications (Vistelius, 1980; Vistelius and Harbaugh, 1980).
3. The calculation of δ_C (as in Tables 7 and 8 in the text) has been added to the program.

PROGRAM LISTING

```
      PROGRAM MARKOV(TAPE5,OUTPUT)                                      1
C                                                                       2
C     ************************************************************      3
C     *                                                          *      4
C     *     THIS FORTRAN PROGRAMME IS A TRANSLATION OF AN ALGOL  *      5
C     * PROGRAMME WHICH ORIGINALLY APPEARED IN VOLUME TWO OF 'IDEAL *      6
C     * GRANITE' BY VISTELIUS & OTHERS, PUBLISHED 1972 BY NAUKA, IN *      7
C     * LENINGRAD, U.S.S.R.                                      *      8
C     *                                                          *      9
C     *                 *          *          *                  *     10
C     *                                                          *     11
C     *     THE PURPOSE OF THIS PROGRAMME IS TO STUDY THE MARKOV *     12
C     * CHAIN PROPERTY FOR THE OCCURRENCES OF MINERALS IN GRANITE.*     13
C     *                                                          *     14
C     *     THIS ANALYSIS IS ACCOMPLISHED BY FIRST BUILDING A TABLE *     15
C     * OF CHI-SQUARE PROBABILITIES IN THE MAIN PROGRAMME, AND THEN *     16
C     * CALLING THREE SUBROUTINES, P0042(), MRKOV(), AND MRKOV2(). *     17
C     *     THE SUBROUTINE P0042() READS IN THE RAW DATA (CHAINS OF *     18
C     * EVENTS), AND PRINTS THEM OUT IN AN EASILY READABLE FORMAT. *     19
C     *     THE SUBROUTINE MRKOV() ANALYSES THESE CHAINS AND GENER- *     20
C     * ATES FIRST, SECOND, AND THIRD ORDER TRANSITION MATRICES AND *     21
C     * TALLY MATRICES.  AS WELL, A SERIES OF STATISTICAL SIGNIF-  *     22
C     * ICANCE TESTS IS PERFORMED.                               *     23
C     *     THE SUBROUTINE MRKOV2() PERFORMS VERIFICATION OF THE *     24
C     * PROPERTIES OF IDEAL GRANITE.                             *     25
C     *                                                          *     26
C     *          FOR INFORMATION ON HOW TO RUN THIS PROGRAMME,   *     27
C     *          PLEASE REFER TO THE PRECEDING USERS' GUIDE.     *     28
C     *                                                          *     29
C     ************************************************************     30
C                                                                       31
C                                                                       32
      DIMENSION ITTL(8),BBARR(100),ABARR(100),CBARR(100),DBARR(100),    33
     + DUMMY1(3,3),DUMMY2(3)                                            34
      COMMON/BLK/ L1(100),IB(3000)                                      35
      COMMON/LABEL1/ DUMMY1,DUMMY2,BBARR,ABARR,CBARR,DBARR              36
      INTEGER S,DUMMY1                                                  37
      REAL NBAR                                                         38
C                                                                       39
      DATA ABARR/0.0039,0.103,0.352,0.711,1.15,1.64,2.17,2.73,3.33,3.94,40
     + 4.57,5.23,5.89,6.57,7.26,7.96,8.67,9.39,10.1,10.9,               41
     + 11.6,12.3,13.1,13.8,14.6,15.4,16.2,16.9,17.7,18.5,               42
     + 19.3,20.1,20.9,21.7,22.5,23.3,24.1,24.9,25.7,26.5,               43
     + 27.3,28.1,29.0,29.8,30.6,31.4,32.3,33.1,33.9,34.8,               44
     + 35.6,36.4,37.3,38.1,39.0,39.8,40.6,41.5,42.3,43.2,               45
     + 44.0,44.9,45.7,46.6,47.4,48.3,49.2,50.0,50.9,51.7,               46
     + 52.6,53.5,54.3,55.2,56.1,56.9,57.8,58.7,59.5,60.4,               47
     + 61.3,62.1,63.0,63.9,64.7,65.6,66.5,67.4,68.2,69.1,               48
     + 70.0,70.9,71.8,72.6,73.5,74.4,75.3,76.2,77.0,77.9/               49
```

```
C                                                                       50
      DATA BBARR/3.84,5.99,7.81,9.49,11.1,12.6,14.1,15.5,16.9,18.3,     51
     + 19.7,21.03,22.4,23.7,25.0,26.3,27.6,28.9,30.1,31.4,              52
     + 32.7,33.9,35.2,36.4,37.7,38.9,40.1,41.3,42.6,43.8,               53
     + 45.0,46.2,47.4,48.6,49.8,51.0,52.2,53.4,54.6,55.8,               54
     + 56.9,58.1,59.3,60.5,61.7,62.8,64.0,65.2,66.3,67.5,               55
     + 68.7,69.8,71.0,72.2,73.3,74.5,75.6,76.8,77.9,79.1,               56
     + 80.2,81.4,82.5,83.7,84.8,86.0,87.1,88.3,89.4,90.5,               57
     + 91.7,92.8,93.9,95.1,96.2,97.4,98.5,99.6,100.7,101.9,             58
     + 103.,104.1,105.3,106.4,107.5,108.6,109.8,110.9,112.,113.1,       59
     + 114.3,115.4,116.5,117.6,118.9,119.9,121.,122.1,123.2,124.3/      60
C                                                                       61
      DATA CBARR/0.000157,0.0201,0.115,0.297,0.554,0.872,1.24,1.65,     62
     + 2.09,2.56,3.05,3.57,4.11,4.66,5.23,5.81,6.41,7.01,7.63,8.26,     63
     + 8.90,9.54,10.2,10.9,11.5,12.2,12.9,13.6,14.3,15.00,              64
     + 15.7,16.4,17.1,17.8,18.5,19.2,20.0,20.7,21.4,22.2,               65
     + 22.9,23.7,24.4,25.1,25.9,26.7,27.4,28.2,28.9,29.7,               66
     + 30.5,31.2,32.0,32.8,33.6,34.3,35.1,35.9,36.7,37.5,               67
     + 38.3,39.1,39.9,40.6,41.4,42.2,43.0,43.8,44.6,45.4,               68
     + 46.2,47.1,47.9,48.7,49.5,50.3,51.1,51.9,52.7,53.5,               69
     + 54.4,55.2,56.0,56.8,57.6,58.5,59.3,60.1,60.9,61.8,               70
     + 62.6,63.4,64.2,65.1,65.9,66.7,67.6,68.4,69.2,70.1/               71
C                                                                       72
      DATA DBARR/6.63,9.21,11.3,13.3,15.1,16.8,18.5,20.1,21.7,23.2,     73
     + 24.7,26.2,27.7,29.1,30.6,32.0,33.4,34.8,36.2,37.6,               74
     + 38.9,40.3,41.6,43.0,44.3,45.6,47.0,48.3,49.6,50.9,               75
     + 52.2,53.5,54.8,56.1,57.3,58.6,59.9,61.2,62.4,63.7,               76
     + 65.0,66.2,67.5,68.7,70.0,71.2,72.4,73.7,74.9,76.2,               77
     + 77.4,78.6,79.8,81.1,82.3,83.5,84.7,86.0,87.2,88.4,               78
     + 89.6,90.8,92.0,93.2,94.4,95.6,96.8,98.0,99.2,100.4,              79
     + 101.6,102.8,104.,105.2,106.4,107.6,108.8,110.,111.1,112.3,       80
     + 113.5,114.7,115.9,117.1,118.2,119.4,120.6,121.8,122.9,124.1,     81
     + 125.3,126.5,127.6,128.8,130.,131.1,132.3,133.5,134.6,135.8/      82
C                                                                       83
C     NOTICE THAT ABARR, BBARR, CBARR, AND DBARR                        84
C       CONTAIN THE  0.05, 0.95, 0.01, AND 0.99 VALUES RESPECTIVELY.    85
C                                                                       86
9     S=3                                                               87
      READ(5,10) ITTL                                                   88
      IF(EOF(5).NE.0) GO TO 30                                          89
10    FORMAT(8A10)                                                      90
      PRINT 20,ITTL                                                     91
20    FORMAT("1",8A10)                                                  92
      PRINT 21                                                          93
21    FORMAT("0", " NUMBER  SUBSEQUENCE"/1X, 3X,"OF",9X,"OF"/           94
     + 1X, " GRAINS",5X,"GRAINS")                                       95
      CALL P0042(L,LBAR,LMAX,LMIN)                                      96
      NBAR=LBAR                                                         97
      LMIN=LMIN-2                                                       98
      LMINS=LMIN*S                                                      99
      LMAX10=LMAX+10                                                    100
      LSSSS=S*S*S*S                                                     101
      CALL MRKOV(S,LMINS,LMAX10,LSSSS,L,NBAR)                           102
      CALL MRKOV2(NBAR)                                                 103
30    STOP                                                              104
      END                                                               105

C                                                                       106
      SUBROUTINE P0042(LCHN,LNN,LMAX,LMIN)                              107
C                                                                       108
C     ********************************************************************* 109
C     *                                                               * 110
C     *     THIS SUBROUTINE USES CARD-FORMAT TO INPUT THE DATA, THEN * 111
C     * PRINTS THEM OUT, GROUPED ACCORDING TO SUBSEQUENCE.  A COUNT  * 112
C     * IS MADE OF THE NUMBER OF GRAINS AND NUMBER OF SUBSEQUENCES.  * 113
C     * SHOULD THESE  NUMBERS  BE TOO  LARGE,  A WARNING MESSAGE IS  * 114
C     * PRINTED AND EXECUTION TERMINATED.                             * 115
C     *                                                               * 116
C     * SUBROUTINE P0042() IS CALLED ONLY ONCE BY THE MAIN PROGRAMME.* 117
C     *                                                               * 118
C     ********************************************************************* 119
C                                                                       120
```

```
C     LIST OF (OUTPUT) PARAMETERS:                                          121
C                                                                           122
C     LCHN  - (INTEGER) NUMBER OF SUBSEQUENCES.                             123
C     LNN   - (INTEGER) NUMBER OF GRAINS.                                   124
C     LMAX  - (INTEGER) MAXIMUM VALUE OVER ALL L1(I),                       125
C                       WHERE L1(I) IS THE NUMBER OF OBSERVATIONS           126
C                             IN THE 'I'TH SUBSEQUENCE.                     127
C     LMIN  - (INTEGER) THE SMALLEST OF ALL L1(I).                          128
C                                                                           129
C     MAXIMUM OF 40 INPUT CARDS                                             130
C     MAXIMUM OF MARKOV CHAIN = 400                                         131
C     MAXIMUM NUMBER OF CHAINS =100                                         132
C     TOTAL NUMBER OF EVENTS NOT TO EXCEED 3000                             133
C                                                                           134
      COMMON/BLK/ L1(100),IB(3000)                                          135
      DIMENSION IAA(320),IBB(80),ICC(400),IFMT(3),IDD(81)                   136
      DATA IFMT/1H(,2H00,7HX,80I1)/                                         137
      DATA IDD/2H00,2H01,2H02,2H03,2H04,2H05,2H06,2H07,2H08,2H09,2H10,      138
     1  2H11,2H12,2H13,2H14,2H15,2H16,2H17,2H18,2H19,2H20,2H21,2H22,2H23139
     2,2H24,2H25,2H26,2H27,2H28,2H29,2H30,2H31,2H32,2H33,2H34,2H35,2H36,140
     32H37,2H38,2H39,2H40,2H41,2H42,2H43,2H44,2H45,2H46,2H47,2H48,2H49, 141
     42H50,2H51,2H52,2H53,2H54,2H55,2H56,2H57,2H58,2H59,2H60,2H61,2H62, 142
     42H63,2H64,2H65,2H66,2H67,2H68,2H69,2H70,2H71,2H72,2H73,2H74,2H75, 143
     5 2H76,2H77,2H78,2H79,2H80/                                          144
C                                                                         145
      KN=0                                                                146
      DO 500 IN=1,320,8                                                   147
      READ(5,600) (IAA(IN+JN-1),JN=1,8)                                   148
      IF(EOF(5)) 605,500                                                  149
500   KN=KN+1                                                             150
600   FORMAT(8A10)                                                        151
C     KN = NUMBER OF INPUT CARDS                                          152
605   KNN=0                                                               153
      LNN=0                                                               154
      LCHN=0                                                              155
      L=0                                                                 156
      LMIN=99999                                                          157
      LMAX=0                                                              158
C                                                                         159
C                                                                         160
610   IL=1                                                                161
      ITEST=0                                                             162


550   KNN=KNN+1                                                           163
      IF(KNN .GT. KN) GO TO 999                                           164
      NC=1                                                                165
      DECODE(80,620,IAA((KNN-1)*8+1)) IBB                                 166
620   FORMAT(80A1)                                                        167
C                                                                         168
C                                                                         169
555   MM=0                                                                170
625   DO 630 IN=NC,80                                                     171
      IF(IBB(IN) .LT. 1H0 .OR. IBB(IN) .GT. 1H9) GO TO 640                172
630   MM=MM+1                                                             173
      GO TO 645                                                           174
640   NC=NC+1                                                             175
      ITEST=1                                                             176
      IF(MM .GT. 0) GO TO 645                                             177
      IF(IL .GT. 1) GO TO 665                                             178
      ITEST=0                                                             179
      IF(NC .GT. 80) GO TO 550                                            180
      GO TO 625                                                           181
645   ILL=IL+MM-1                                                         182
      LNN=LNN+MM                                                          183
      IFMT(2)=IDD(NC-ITEST)                                               184
      DECODE(80,IFMT,IAA((KNN-1)*8+1))  (ICC(IN),IN=IL,ILL)               185
660   IL=ILL +1                                                           186
      NC=NC+MM                                                            187
      IF(ITEST .EQ. 0) GO TO 550                                          188
```

```
665   PRINT 670, ILL                                                    189
      PRINT 700, (ICC(IN),IN=1,ILL)                                     190
      DO 666 IN=1,ILL                                                   191
      L=L+1                                                             192
666   IB(L)=ICC(IN)                                                     193
670   FORMAT(" ", "  (",I3,")" )                                        194
700   FORMAT(1H+, 9X,120I1/(1X, 9X,120I1))                              195
      LCHN=LCHN+1                                                       196
      L1(LCHN)=ILL                                                      197
      IF(LMIN .GT. ILL) LMIN=ILL                                        198
      IF(LMAX .LT. ILL) LMAX=ILL                                        199
710   IF(NC .GT. 80) GO TO 610                                          200
      IL=1                                                              201
      ITEST=0                                                           202
      GO TO 555                                                         203
999   IF(IL .EQ. 1) GO TO 9999                                          204
      PRINT 700, (ICC(IN),IN=1,ILL)                                     205
      LCHN=LCHN+1                                                       206
      DO 725 IN=1,ILL                                                   207
      L=L+1                                                             208
725   IB(L)=ICC(IN)                                                     209
      L1(LCHN)=ILL                                                      210
      IF(LMIN .GT. ILL) LMIN=ILL                                        211
      IF(LMAX .LT. ILL) LMAX=ILL                                        212
C9999 IF(KTEST.GT.0) GO TO 730                                          213
9999  PRINT 800, LCHN,LNN                                               214
800   FORMAT("0", " NUMBER OF SUBSEQUENCES = ",I4/                      215
     +  "0", " TOTAL NUMBER OF GRAINS = ",I4)                           216
      IF(KN.GT.39) PRINT 810                                            217
      IF(LCHN.GT.100) GO TO 860                                         218
      IF(LNN.GT.3000) GO TO 870                                         219
      DO 830 I=1,LCHN                                                   220
      IF(L1(I).GT.400) GO TO 880                                        221
830   CONTINUE                                                          222
      RETURN                                                            223
860   PRINT 820, LCHN                                                   224
      GO TO 890                                                         225
870   PRINT 840, LNN                                                    226
      GO TO 890                                                         227
880     PRINT 850, L1(I),I                                              228
890   STOP                                                              229
810   FORMAT("0","****WARNING   THIS PROGRAMME WILL ONLY PROCESS",      230
     + " A MAXIMUM OF 40 INPUT DATA CARDS"/16X,"CHECK THE LISTING OF",  231
     + " DATA TO MAKE SURE THAT ALL DATA ARE PROCESSED")                232
820   FORMAT("0","****WARNING   THIS PROGRAMME WILL ONLY PROCESS",      233
     + " A MAXIMUM OF 100 MARKOV CHAINS AT A TIME, BUT YOUR DATA ",     234
     + "CONTAIN ",I4," MARKOV CHAINS.")                                 235
840   FORMAT("0","****WARNING   THIS PROGRAMME WILL ONLY PROCESS",      236
     + " A MAXIMUM OF 3000 EVENTS IN A MARKOV EXPERIMENT, BUT YOUR",    237
     + " DATA CONTAIN ",I5," EVENTS.")                                  238
850   FORMAT("0","****WARNING   THIS PROGRAMME WILL PROCESS A ",        239
     + "MAXIMUM OF 400 EVENTS IN ANY MARKOV CHAIN, BUT YOUR DATA ",     240
     + "CONTAIN ",I4," EVENTS"/16X, "IN THE ",I4,"TH CHAIN.")           241
      END                                                               242

      SUBROUTINE MRKOV(S,LMINS,LMAX10,LSSSS,L,NBAR)                     243
C                                                                       244
C     ********************************************************************* 245
C     *                                                                   * 246
C     *     THIS SUBROUTINE  GIVES  TRANSITIONAL FREQUENCIES,  AND        * 247
C     * COMPUTES INITIAL AND TRANSITIONAL PROBABILITIES.   IT ALSO        * 248
C     * TESTS THE REVERSIBILITY OF THE GIVEN INPUT SEQUENCE, TESTS        * 249
C     * FOR THE ORDER OF MARKOVIAN BEHAVIOUR, AND TESTS HYPOTHESES        * 250
C     * OF HOMOGENEITY.                                                  * 251
C     *                                                                   * 252
C     *     SUBROUTINE MRKOV()  IS CALLED  ONLY ONCE  BY THE MAIN         * 253
C     * PROGRAMME,  AND IN TURN,  MAKES AS MANY AS  FOUR CALLS TO         * 254
C     * SUBROUTINE CHISQR().                                              * 255
C     *                                                                   * 256
C     ********************************************************************* 257
C                                                                         258
```

```
C     S     - (INTEGER) NUMBER OF DIFFERENT POSSIBLE STATES                  259
C     LMINS - (INTEGER) MINIMUM OVER ALL L1(I), LESS 2,                      260
C                       AND MULTIPLIED BY 'S'.                               261
C     LMAX10- (INTEGER) MAXIMUM OVER ALL L1(I), INCREASED BY 10.             262
C     LSSSS - (INTEGER) 'S' TO THE POWER 4.                                  263
C     L     - (INTEGER) NUMBER OF SUBSEQUENCES OR CHAINS.                    264
C     NBAR  - (REAL) NUMBER OF GRAINS OR 'EVENTS'                            265
C                                                                            266
C     L1(K) - LENGTH OF THE 'K'TH CHAIN.                                     267
C     A( )  - SEQUENCE OF EVENTS IN THE CHAIN.                               268
C                                                                            269
C     BBAR( ,I2) = NUMBER OF OCCURRENCES OF  I1-**-I2                        270
C                                            I1-**-**-I2                     271
C                                            I1-**-**-**-I2                  272
C                                            I1-**-**-**-**-I2               273
C                                            I1-**-**-**-**-**-I2            274
C                                            I1-**-**-**-**-**-**-I2         275
C                = (REDEFINED) PROBABILITY FOR THE OCCURRENCE OF K,          276
C                    DUE TO ALL THE PRECEDING EVENTS.                        277
C                                                                            278
      COMMON/BLK/ L1(100),IB(3000)                                           279
      COMMON/LABEL1/ SUMN2,SUML,BBARR                                        280
      DIMENSION MP(3,3),MP1(3,3),MP2(3,3),P(3),SUMN2(3,3),A(400),            281
     + B(400),BBAR(400,3),SUMN3(3,3,3),SUMN4(81),SUMS(3),N2(3,3),            282
     + N3(3,3,3),N4(81),N1(3),NK(3),BBARR(100)                               283
      DIMENSION IS(6),SUML(3)                                                284
      INTEGER S,T,Q,SUMN2,SUMN3,SUMN4,SUMS,A,B,ABAR(120),MEMORY(3000)         285
      REAL MP,MP1,MP2,NBAR                                                   286
C                                                                            287
C                                                                            288
      LMAX=LMAX10-10                                                         289
      LMIN=LMINS/S                                                           290
      Q=0                                                                    291
C                                                                            292
      DO 10 I1=1,S                                                           293
C                                                                            294
C     SUML ARRAY INSERTED BY K F CHONG                                       295
C                                                                            296
      SUML(I1)=0.0                                                           297
      NK(I1)=SUMS(I1)=0                                                      298
      DO 10 I2=1,S                                                           299
      SUMN2(I1,I2)=0                                                         300
      DO 10 I3=1,S                                                           301
      SUMN3(I1,I2,I3)=0                                                      302
10    CONTINUE                                                               303
      DO 11 I4=1,LSSSS                                                       304
11    SUMN4(I4)=0                                                            305
      DO 20 I1=1,LMINS                                                       306
      B(I1)=0                                                                307
      DO 20 I2=1,S                                                           308
20    BBAR(I1,I2)=0                                                          309
C                                                                            310
C     *********   START OF LOOP   *********                                  311
C                                                                            312
C                                                                            313
C     DATA INITIALISATION                                                    314
C                                                                            315
      LU=0                                                                   316
      DO 30 K=1,L                                                            317
C     ANALYSE EACH MARKOV CHAIN, SAY THE K-CHAIN.                            318
C     LK10=L1(K)+10                                                          319
C     DO 25 T=1,LK10                                                         320
      DO 25 T=1,LMAX10                                                       321
25    A(T)=0                                                                 322
      DO 40 I1=1,S                                                           323
      N1(I1)=0                                                               324
      DO 40 I2=1,S                                                           325
      N2(I1,I2)=0                                                            326
      DO 40 I3=1,S                                                           327
      N3(I1,I2,I3)=0                                                         328
40    CONTINUE                                                               329
      DO 41 I4=1,LSSSS                                                       330
41    N4(I4)=0                                                               331
C                                                                            332
C                                                                            333
```

```
C     DATA INPUT ---  ARRAY A.                                          334
C                                                                       335
C                                                                       336
C                                                                       337
      LU=LU+L1(K)                                                       338
      LL=LU-L1(K)+1                                                     339
      J=0                                                               340
      DO 26 IN=LL,LU                                                    341
      J=J+1                                                             342
26    A(J) =IB(IN)                                                      343
29    LK1=L1(K)-1                                                       344
      DO 50 J=1,LK1                                                     345
      I1=A(J)                                                           346
      I2=A(J+1)                                                         347
      I3=A(J+2)                                                         348
      I4=A(J+3)                                                         349
42    IF(J .GT. (LK1-2)) GO TO 43                                       350
C     COUNTING THE OCCURRENCES OF THE CHAIN  ---                        351
C                                  ****** I1-I2-I3-I4                 352
C                                  ****** I1-I2-I3                    353
C                                  ****** I1-I2                       354
C                                                                       355
C     ARRAY(I1,I2,I3,I4)=ARRAY((I1-1+K*(I2-1+L*(I3-1+M*(I4-1)))))      356
      IIII=I1+S*(I2-1+S*(I3-1+S*(I4-1)))                                357
      N4(IIII)=N4(IIII)+1                                               358
      SUMN4(IIII)=SUMN4(IIII)+1                                         359
43    IF(J .GT. (LK1-1)) GO TO 44                                       360
      N3(I1,I2,I3)=N3(I1,I2,I3)+1                                       361
      SUMN3(I1,I2,I3)=SUMN3(I1,I2,I3)+1                                 362
44    N2(I1,I2)=N2(I1,I2)+1                                             363
      SUMN2(I1,I2)=SUMN2(I1,I2)+1                                       364
      N1(I1)=N1(I1)+1                                                   365
      SUMS(I1)=SUMS(I1)+1                                               366
50    CONTINUE                                                          367
C                                                                       368
C                                                                       369
      N=L1(K)                                                           370
      I1=A(N)                                                           371
      SUML(I1)=SUML(I1)+1.0                                             372
C                                                                       373
C     NK(I1) = OCCURRENCE OF I1 AT THE END OF CHAIN.                    374
      NK(I1)=NK(I1)+1                                                   375
      SUMS(I1)=SUMS(I1)+1                                               376
      N1(I1)=N1(I1)+1                                                   377
      DO 60 K1=1,S                                                      378
C     NUMBER OF OCCURRENCES OF     K1 IN THE CHAIN.                     379
      ABAR(K1)=N1(K1)                                                   380
      DO 60 K2=1,S                                                      381
C     NUMBER OF OCCURRENCES OF     K1-K2 IN THE CHAIN.                  382
      ABAR(K1*S+K2)=N2(K1,K2)                                           383
      DO 60 K3=1,S                                                      384
C     NUMBER OF OCCURRENCES OF     K1-K2-K3 IN THE CHAIN.               385
      ABAR(K1*S*S+K2*S+K3)=N3(K1,K2,K3)                                 386
      DO 60 K4=1,S                                                      387
      KKKK=K1+S*(K2-1+S*(K3-1+S*(K4-1)))                                388
C     NUMBER OF OCCURRENCES OF     K1-K2-K3-K4 IN THE CHAIN.            389
      ABAR(K1*(S**3)+K2*S*S+K3*S+K4)=N4(KKKK)                           390
60    CONTINUE                                                          391
C                                                                       392
C                                                                       393
      Q=Q+1                                                             394
C                                                                       395
C     STORE ABAR                                                        396
C                                                                       397
C                                                                       398
C     THIS PORTION OF THE PROGRAMME IS NEVER USED.                      399
C                                                                       400
C     --------                                                          401
      IBEGIN=(K-1)*LSSSS+1                                              402
      IEND=K*LSSSS+1                                                    403
      DO 61 INDEX=IBEGIN,IEND                                           404
      MEMORY(INDEX)=ABAR(INDEX-(K-1)*LSSSS)                             405
```

```
61    CONTINUE                                                          406
C     PRINT 490,ABAR                                                    407
C     EFFECT OF THE PRECEDING EVENTS.                                   408
      DO 70 N=1,LMIN                                                    409
      L1KN=L1(K)-N-1                                                    410
      DO 70 M=1,L1KN                                                    411
      I1=A(M)                                                           412
      I2=A(M+N+1)                                                       413
      J=(N-1)*S+I1                                                      414
C     NUMBER OF OCCURRENCES OF  ---  I1-**-I2                           415
C     NUMBER OF OCCURRENCES OF  ---  I1-**--**-I2                       416
C     NUMBER OF OCCURRENCES OF  ---  I1-**--**-**-I2                    417
      BBAR(J,I2)=BBAR(J,I2)+1                                           418
70    CONTINUE                                                          419
C     --------                                                          420
C                                                                       421
C                                                                       422
30    CONTINUE                                                          423
C                                                                       424
C     *********   END OF THE LOOP   *********                           425
C                                                                       426
C     ALL THE MARKOV CHAINS HAVE BEEN ANALYSED.                         427
C                                                                       428
C                                                                       429
C                                                                       430
C                                                                       431
C  - - - - - - - - - - - - - - - - - - - -                              432
C     PRINT OUT THE RESULTS OF THIS ANALYSIS                            433
C                                                                       434
      PRINT 63                                                          435
63    FORMAT("1", 32X,"STATE 1",5X,"STATE 2",5X,"STATE 3"/              436
     + 1X, "NUMBER OF SUBSEQUENCES")                                    437
      PRINT 64, NK                                                      438
64    FORMAT(" ", 13X,"ENDING WITH",3I12)                               439
      PRINT 65                                                          440
65    FORMAT("0"," FREQUENCIES"//1X,"   FROM STATE(S)")                 441
      PRINT 66, SUMS                                                    442
66    FORMAT("0", 5X,"1 GRAIN -N(I)"/1X, 9X,"F(T)=",11X,3I12)           443
      PRINT 59, SUMS(1)-NK(1),SUMS(2)-NK(2),SUMS(3)-NK(3)               444
59    FORMAT(" ", 9X,"F(P)=",11X,3I12)                                  445
      PRINT 67                                                          446
67    FORMAT("0", 5X, "2 GRAINS-N(I;J)")                                447
      DO 71 IX=1,S                                                      448
71    PRINT 68, IX, (SUMN2(IX,IY),IY=1,S)                               449
68    FORMAT(1X, 24X,I1,8I12)                                           450
      PRINT 69                                                          451
69    FORMAT("0", 5X,"3 GRAINS-N(I,J;K)")                               452
      DO 72 IX=1,S                                                      453
      DO 72 IY=1,S                                                      454
      PRINT 76, IX, IY, (SUMN3(IX,IY,IZ),IZ=1,S)                        455
76    FORMAT(1X, 23X,2I1,8I12)                                          456
72    CONTINUE                                                          457
      PRINT 77                                                          458
77    FORMAT("0", 5X,"4 GRAINS-N(I,J,K;L)")                             459
      DO 74 IX=1,S                                                      460
      DO 74 IY=1,S                                                      461
      DO 74 IZ=1,S                                                      462
      DO 73 IW=1,S                                                      463
73    IS(IW)=IX+S*(IY-1+S*(IZ-1+S*(IW-1)))                              464
74    PRINT 78, IX,IY,IZ,SUMN4(IS(1)),SUMN4(IS(2)),                     465
     +  SUMN4(IS(3))                                                    466
78    FORMAT(1X, 22X,3I1,3I12)                                          467
C                                                                       468
C                                                                       469
C -------------------------------------------------------------         470
      DO 80 N=1,LMINS                                                   471
      DO 75 M=1,S                                                       472
75    B(N)=B(N)+BBAR(N,M)                                               473
      DO 80 K=1,S                                                       474
C     PROBABILITY FOR THE OCCURRENCE OF K DUE TO ALL THE                475
C       PRECEDING EVENTS.                                               476
      BBAR(N,K)=BBAR(N,K)/B(N)                                          477
80    CONTINUE                                                          478
C                                                                       479
C                                                                       480
```

```
      DO 82 I=1,S                                                       481
C     PROBABILITY FOR THE OCCURRENCE OF I.                              482
      P(I)=FLOAT(SUMS(I))/NBAR                                          483
C                                                                       484
C     EXCLUDE THE OCCURRENCE OF I AT THE END OF CHAIN.                  485
C                                                                       486
      SUMS(I)=SUMS(I)-NK(I)                                             487
      DO 82 J=1,S                                                       488
C                                                                       489
C     PROBABILITY OF THE EVENT I-J                                      490
C        DUE TO THE TOTAL NUMBER OF EVENTS, I.                          491
      MP1(I,J)=FLOAT(SUMN2(I,J))/FLOAT(SUMS(I))                         492
      MP(I,J)=MP1(I,J)                                                  493
82    CONTINUE                                                          494
C                                                                       495
C  - - - - - - - - - - - - - - - - -  PRINT THE RESULTS                 496
C                                                                       497
      PRINT 84                                                          498
84    FORMAT("1", 14X,"INITIAL PROBABILITIES - IP(I)")                  499
      PRINT 85                                                          500
85    FORMAT(" ", 15X,"(1)",9X,"(2)",9X,"(3)")                          501
      PRINT 86, P                                                       502
86    FORMAT(" ", 9X,3F12.8)                                            503
      PRINT 89                                                          504
89    FORMAT("-", 9X,"TRANSITIONAL PROBABILITIES - TP(I,J)")             505
      PRINT 85                                                          506
      DO 91 IX=1,S                                                      507
91    PRINT 94, IX,(MP(IX,IY),IY=1,S)                                   508
94    FORMAT(" ", 6X,"(",I1,")",3F12.8)                                 509
      K=1                                                               510
C                                                                       511
C ----------------------------------------------------------------      512
C                                                                       513
95    DO 100 I=1,S                                                      514
      DO 100 J=1,S                                                      515
      MP2(I,J)=0                                                        516
      DO 100 T=1,S                                                      517
C                                                                       518
C     PROBABILITY OF THE EVENT I-**-J.                                  519
      MP2(I,J)=MP2(I,J)+MP(I,T)*MP1(T,J)                                520
100   CONTINUE                                                          521
      K=K+1                                                             522
C                                                                       523
C                                                                       524
      DO 105 I=1,S                                                      525
      DO 105 J=1,S                                                      526
105   MP1(I,J)=MP2(I,J)                                                 527
      M=MP2(2,3)*1.0E8                                                  528
      N=MP2(3,3)*1.0E8                                                  529
      IF(M .EQ. N) GO TO 120                                            530
      IF(K .LT. 30) GO TO 95                                            531
C                                                                       532
C ----------------------------------------------------------------      533
C     CALCULATION OF TEST-VALUES FOR REVERSIBILITY                      534
C                                                                       535
120   H1=H2=H3=0                                                        536
      DO 110 II=1,S                                                     537
      DO 110 JJ=1,S                                                     538
      H=SUMN2(II,JJ)-SUMN2(JJ,II)                                       539
      U=SUMN2(II,JJ)+SUMN2(JJ,II)                                       540
      IF(U .EQ. 0) GO TO 130                                            541
      R=H*H/U                                                           542
      H1=H1+R/2                                                         543
C                                                                       544
130   DO 110 KK=1,S                                                     545
      H=SUMN3(II,JJ,KK)-SUMN3(KK,JJ,II)                                 546
      U=SUMN3(II,JJ,KK)+SUMN3(KK,JJ,II)                                 547
      IF(U .EQ. 0) GO TO 140                                            548
      R=H*H/U                                                           549
      H2=H2+R/2                                                         550
C                                                                       551
```

```
140   DO 110 I1=1,S                                                    552
      IIII=II+S*(JJ-1+S*(KK-1+S*(I1-1)))                               553
      JJJJ=I1+S*(KK-1+S*(JJ-1+S*(II-1)))                               554
      H=SUMN4(IIII)-SUMN4(JJJJ)                                        555
      U=SUMN4(IIII)+SUMN4(JJJJ)                                        556
      IF(U .EQ. 0) GO TO 110                                           557
      R=H*H/U                                                          558
      H3=H3+R/2                                                        559
110   CONTINUE                                                         560
C                                                                      561
C  - - - - - - - - - - - - - - - - -  PRINT THE RESULTS                 562
C                                                                      563
      PRINT 141                                                        564
141   FORMAT("-", "STATISTICAL SIGNIFICANCE TESTS"/"0", 40X,           565
     + "0TH ORDER    1ST ORDER    2ND ORDER    3RD ORDER")             566
      PRINT 142, H1,H2,H3                                              567
142   FORMAT("0", "REVERSIBILITY  (CHI-SQUARED)",21X,3F12.4)           568
C                                                                      569
C     GET THE APPROPRIATE CHI-SQUARE VALUES.                           570
C                                                                      571
C     USING THE FORMULA FOR CALCULATING THE NUMBER OF DEGREES          572
C       OF FREEDOM, NU, FOR REVERSIBILITY, WE FIND THAT                573
C         NU1=3, NU2=9, & NU3=36,  ALWAYS.                             574
C                                                                      575
      PRINT 143, BBARR(3),BBARR(9),BBARR(36)                           576
143   FORMAT(" ", "THEORETICAL CHI-SQUARE VALUES AT 0.05",12X,3F12.4)  577
      PRINT 144                                                        578
144   FORMAT(" ", "NUMBER OF DEGREES OF FREEDOM",29X,"3",11X,          579
     + "9",11X,"36")                                                  580
C                                                                      581
C ---------------------------------------------------------------------582
C     CALCULATION OF TEST-VALUES FOR MARKOVITY                         583
C                                                                      584
      H=0                                                              585
      DO 150 I=1,S                                                     586
150   N1(I)=ABAR(I)=NK(I)=0                                            587
      DO 160 I=1,S                                                     588
      DO 160 J=1,S                                                     589
C                                                                      590
C     NK(J) = ALL THE OCCURRENCES OF **-J.                             591
      NK(J)=NK(J)+SUMN2(I,J)                                           592
      N2(I,J)=MP(I,J)=MP1(I,J)=MP2(I,J)=0                              593
160   CONTINUE                                                         594
      CBAR=NBAR-FLOAT(L)                                               595
C                                                                      596
C  - - - - - - - - - - - - - - - - - - - -                             597
C     FIRST ORDER LAMBDA -    GET H1                                   598
C                                                                      599
C     NBAR = TOTAL CASES                                               600
      DO 170 I=1,S                                                     601
      DO 170 J=1,S                                                     602
      IF(SUMN2(I,J) .EQ. 0) GO TO 170                                  603
C                                                                      604
C     FORMULA REVISED BY K F CHONG                                     605
C     REPLACE NBAR BY CBAR                                             606
C                                                                      607
C     R=FLOAT(SUMN2(I,J))*NBAR/(FLOAT(SUMS(I))*FLOAT(NK(J)))           608
      R=FLOAT(SUMN2(I,J))*CBAR/(FLOAT(SUMS(I))*FLOAT(NK(J)))           609
      H=H+SUMN2(I,J)*ALOG(R)                                           610
170   CONTINUE                                                         611
C                                                                      612
C     FIRST ORDER                                                      613
      H1=2*H                                                           614
      IF(H1 .GE. 9.49) GO TO 175                                       615
      PRINT 171, H1                                                    616
171   FORMAT("0", "MARKOVITY",4X,"(CHI-SQUARE)",24X,3F12.4)            617
      NU1=4                                                            618
      PRINT 143, BBARR(NU1)                                            619
      PRINT 172, NU1                                                   620
172   FORMAT(" ", "NUMBER OF DEGREES OF FREEDOM",19X,3I12)             621
      GO TO 215                                                        622
C                                                                      623
C                                                                      624
```

```
C                                   GET H2                                  625
175   DO 180 I=1,S                                                          626
      DO 180 J=1,S                                                          627
      M=I*S*S+J*S                                                           628
      DO 180 K=1,S                                                          629
      ABAR(M+K)=N3(I,J,K)=0                                                 630
      MP1(I,J)=MP1(I,J)+SUMN3(I,J,K)                                        631
      N2(J,K)=N2(J,K)+SUMN3(I,J,K)                                          632
      N1(J)=N1(J)+SUMN3(I,J,K)                                              633
180   CONTINUE                                                              634
      H=0                                                                   635
      DO 190 I=1,S                                                          636
      DO 190 J=1,S                                                          637
      DO 190 K=1,S                                                          638
      IF(SUMN3(I,J,K) .EQ. 0) GO TO 190                                     639
      R=FLOAT(SUMN3(I,J,K))*FLOAT(N1(J))/( MP1(I,J)*N2(J,K))                640
      MP2(J,J)=MP2(J,J)+2*SUMN3(I,J,K)*ALOG(R)                              641
190   CONTINUE                                                              642
      R1=MP2(1,1)                                                           643
      R2=MP2(2,2)                                                           644
      R3=MP2(3,3)                                                           645
C     SAVE THESE VALUES FOR PRINTOUT LATER                                  646
      ALT1=R1                                                               647
      ALT2=R2                                                               648
      ALT3=R3                                                               649
      R4=R1+R2                                                              650
C                                                                           651
C     SECOND ORDER                                                          652
      H2=R4+R3                                                              653
      IF(H2 .GE. 21.03) GO TO 195                                           654
      PRINT 171, H1,H2                                                      655
      NU1=4                                                                 656
      NU2=12                                                                657
      PRINT 143, BBARR(NU1),BBARR(NU2)                                      658
      PRINT 172, NU1,NU2                                                    659
      GO TO 215                                                             660
C                                                                           661
C                                                                           662
C                                   GET H3                                  663
195   DO 200 I=1,S                                                          664
      DO 200 J=1,S                                                          665
      DO 200 K=1,S                                                          666
      K1=J*S*S+K*S                                                          667
      K2=I*(S**3)+K1                                                        668
      DO 200 I1=1,S                                                         669
      IIII=I+S*(J-1+S*(K-1+S*(I1-1)))                                       670
      ABAR(K2+I1)=N4(IIII)=0                                                671
      K3=K1+I1                                                              672
      N3(I,J,K)=N3(I,J,K)+SUMN4(IIII)                                       673
      ABAR(K3)=ABAR(K3)+SUMN4(IIII)                                         674
      MP(J,K)=MP(J,K)+SUMN4(IIII)                                           675
200   CONTINUE                                                              676
C                                                                           677
C                                                                           678
      DO 210 I=1,S                                                          679
      DO 210 J=1,S                                                          680
      DO 210 K=1,S                                                          681
      K1=J*S*S+K*S                                                          682
      DO 210 I1=1,S                                                         683
      K3=K1+I1                                                              684
      IF(ABAR(K3) .EQ. 0) GO TO 210                                         685
      IF(N3(I,J,K) .EQ. 0) GO TO 210                                        686
      IIII=I+S*(J-1+S*(K-1+S*(I1-1)))                                       687
      IF(SUMN4(IIII) .EQ. 0) GO TO 210                                      688
      R=FLOAT(SUMN4(IIII))*MP(J,K)/(ABAR(K3)*N3(I,J,K))                     689
      H=H+SUMN4(IIII)*ALOG(R)                                               690
210   CONTINUE                                                              691
C                                                                           692
C     THIRD ORDER                                                           693
      H3=2*H                                                                694
C                                                                           695
      PRINT 171, H1,H2,H3                                                   696
C                                                                           697
C     USING THE FORMULA   NU(K) = 4 * 3**(K-1),  WE GET                     698
```

```
C       NU1=4, NU2=12, & NU3=36 (DEGREES OF FREEDOM) ALWAYS.
C
      NU1=4
      NU2=12
      NU3=36
      PRINT 143, BBARR(NU1),BBARR(NU2),BBARR(NU3)
      PRINT 172, NU1,NU2,NU3
C
215   R1=R2=R3=R4=0
C
C ---------------------------------------------------------------
C     CALCULATION OF TEST-VALUES FOR HOMOGENEITY
C
      Q=L+1
      DO 325 IQ=1,L
      Q=Q-1
C     CHECK THE CORRELATION FOR THE OCCURRENCES OF  --  I1-I2
C                                                      I1-I2-I3
C                                                      I1-I2-I3-I4
C     CALL P0737(.FALSE.,Q,ABAR)
C
C
C                          GET R1
      IBEGIN=(IQ-1)*LSSSS+1
      IEND=IQ*LSSSS
      DO 217 INDEX=IBEGIN,IEND
      ABAR(INDEX-(IQ-1)*LSSSS)=MEMORY(INDEX)
217   CONTINUE
      K=Q
      DO 220 I=1,S
      NK(I)=0
C
C     K IN THE ORIGINAL VERSION REPLACED BY IQ  --- K F CHONG
C
      U=FLOAT(ABAR(I))/FLOAT(L1(IQ))-P(I)
      H=U*U/P(I)*L1(IQ)
C     U=FLOAT(ABAR(I))/FLOAT(L1(K))-P(I)
C     H=U*U/P(I)*L1(K)
220   R1=R1+H
      IF(H1 .LT. 9.49) GO TO 325
C
C
C                          GET R2
      DO 240 I1=1,S
      DO 230 I2=1,S
230   NK(I1)=NK(I1)+ABAR(I1*S+I2)
      IF(NK(I1) .EQ. 0) GO TO 240
      DO 250 J=1,S
      MP(I1,J)=0
      IF(SUMN2(I1,J) .EQ. 0) GO TO 250
      H=FLOAT(SUMN2(I1,J))/FLOAT(SUMS(I1))
      U=FLOAT(ABAR(I1*S+J))/FLOAT(NK(I1))
      R=U-H
      U=R*R/H
      R2=R2+NK(I1)*U
250   CONTINUE
240   CONTINUE
      IF(H2 .LT. 21.03) GO TO 325
C
C
C                          GET R3
      DO 260 I1=1,S
      DO 260 I2=1,S
      J=I1*S*S+I2*S
      DO 270 I3=1,S
270   MP(I1,I2)=MP(I1,I2)+FLOAT(ABAR(J+I3))
      IF(MP(I1,I2) .EQ. 0) GO TO 260
      IF(MP1(I1,I2) .EQ. 0 ) GO TO 260
      DO 280 I4=1,S
      IF(SUMN3(I1,I2,I4) .EQ. 0) GO TO 260
      R=ABAR(J+I4)/MP(I1,I2)
      U=SUMN3(I1,I2,I4)/MP1(I1,I2)
```

```
      H=R-U                                                              771
      R=H*H/U*MP(I1,I2)                                                  772
      R3=R3+R                                                            773
      ABAR(J+I4)=0                                                       774
280   CONTINUE                                                           775
260   CONTINUE                                                           776
      IF(H3 .LT. 51.0) GO TO 325                                         777
C                                                                        778
C                                                                        779
C                              GET R4                                    780
      DO 290 I1=1,S                                                      781
      DO 290 I2=1,S                                                      782
      I=I1*S*S+I2*S                                                      783
      DO 300 I3=1,S                                                      784
      J=I1*(S**3)+I2*S*S+I3*S                                            785
      DO 310 I4=1,S                                                      786
310   ABAR(I+I3)=ABAR(I+I3)+ABAR(J+I4)                                   787
      IF(ABAR(I+I3) .EQ. 0) GO TO 300                                    788
      IF(N3(I1,I2,I3) .EQ. 0) GO TO 300                                  789
      DO 320 I5=1,S                                                      790
      IIII=I1+S*(I2-1+S*(I3-1+S*(I5-1)))                                 791
      IF(SUMN4(IIII) .EQ. 0) GO TO 320                                   792
      R=FLOAT(ABAR(J+I5))/FLOAT(ABAR(I+I3))                              793
      U=FLOAT(SUMN4(IIII))/FLOAT(N3(I1,I2,I3))                           794
      H=R-U                                                              795
      R=H*H/U*ABAR(I+I3)                                                 796
      R4=R4+R                                                            797
      ABAR(J+I5)=0                                                       798
320   CONTINUE                                                           799
300   CONTINUE                                                           800
290   CONTINUE                                                           801
C                                                                        802
C                                                                        803
325   CONTINUE                                                           804
C                                                                        805
C                                                                        806
C  - - - - - - - - - - - - - - - - -  PRINT THE RESULTS                  807
C                                                                        808
C     CALL P1041(R1)                                                     809
C     CALL P1041(R2)                                                     810
C     CALL P1041(R3)                                                     811
C     CALL P1041(R4)                                                     812
330   FORMAT("0", "HOMOGENEITY",4X,"(CHI-SQUARED)",9X,4F12.4)            813
C                                                                        814
C     THE FORMULA FOR DEGREES-OF-FREEDOM IS GIVEN BY                     815
C          NU(N,L) = 3**N * 2*(L-1)                                      816
C        FOR N = 0 TO 3,     L = NR. OF SUBSEQUENCES                   817
C                                                                        818
      NU1=2*(L-1)                                                        819
      NU2=3*2*(L-1)                                                      820
      NU3=9*2*(L-1)                                                      821
      NU4=27*2*(L-1)                                                     822
C                                                                        823
      IF(H1 .GE. 9.49) GO TO 360                                         824
      PRINT 330, R1                                                      825
      CALL CHISQR(NU1,CHI5,CHI95,CHI1,CHI99)                             826
      XSQR1=CHI95                                                        827
      PRINT 340, XSQR1                                                   828
340   FORMAT(" ", "THEORETICAL CHI-SQUARE VALUES AT 0.05",4F12.4)        829
      PRINT 350, NU1                                                     830
350   FORMAT(" ","NUMBER OF DEGREES OF FREEDOM",7X,4I12)                 831
      GO TO 390                                                          832
C                                                                        833
360   IF(H2 .GE. 21.03) GO TO 370                                        834
      PRINT 330, R1,R2                                                   835
      CALL CHISQR(NU1,CHI5,CHI95,CHI1,CHI99)                             836
      XSQR1=CHI95                                                        837
      CALL CHISQR(NU2,CHI5,CHI95,CHI1,CHI99)                             838
      XSQR2=CHI95                                                        839
      PRINT 340, XSQR1,XSQR2                                             840
      PRINT 350, NU1,NU2                                                 841
      GO TO 390                                                          842
```

```
C                                                                       843
370   IF(H3 .GE. 51.0) GO TO 380                                        844
      PRINT 330, R1,R2,R3                                               845
      CALL CHISQR(NU1,CHI5,CHI95,CHI1,CHI99)                            846
      XSQR1=CHI95                                                       847
      CALL CHISQR(NU2,CHI5,CHI95,CHI1,CHI99)                            848
      XSQR2=CHI95                                                       849
      CALL CHISQR(NU3,CHI5,CHI95,CHI1,CHI99)                            850
      XSQR3=CHI95                                                       851
      PRINT 340, XSQR1,XSQR2,XSQR3                                      852
      PRINT 350, NU1,NU2,NU3                                            853
      GO TO 390                                                         854
C                                                                       855
380   PRINT 330, R1,R2,R3,R4                                            856
      CALL CHISQR(NU1,CHI5,CHI95,CHI1,CHI99)                            857
      XSQR1=CHI95                                                       858
      CALL CHISQR(NU2,CHI5,CHI95,CHI1,CHI99)                            859
      XSQR2=CHI95                                                       860
      CALL CHISQR(NU3,CHI5,CHI95,CHI1,CHI99)                            861
      XSQR3=CHI95                                                       862
      CALL CHISQR(NU4,CHI5,CHI95,CHI1,CHI99)                            863
      XSQR4=CHI95                                                       864
      PRINT 340, XSQR1,XSQR2,XSQR3,XSQR4                                865
      PRINT 350, NU1,NU2,NU3,NU4                                        866
C                                                                       867
390   CONTINUE                                                          868
      PRINT 400                                                         869
400   FORMAT("-", 9X,"ALTERNATIVES FOR PARTIAL TRANSITIONS")            870
      PRINT 85                                                          871
      PRINT 410, ALT1,ALT2,ALT3                                         872
410   FORMAT(" ", 7X,3F12.4)                                            873
      PRINT 420                                                         874
420   FORMAT("0", 4X,"NOTE: FOR THE ABOVE, THE THEORETICAL ",           875
     + "CHI-SQUARED VALUES"/" ", 12X,"ARE ALL 9.49, FOR 4 DEGREES",     876
     + " OF FREEDOM.")                                                  877
C                                                                       878
      RETURN                                                            879
      END                                                               880

      SUBROUTINE MRKOV2(NBAR)                                           881
C                                                                       882
C     ********************************************************************883
C     *                                                                  *884
C     *     SUBROUTINE MRKOV2() CALCULATES AND PRINTS THE MATRIX OF      *885
C     * PARAMETERS, C, AND A MATRIX OF INTERMEDIATE RESULTS, MP4.        *886
C     * THE PROPERTIES OF SYMMETRY AND UNITARITY ARE VERIFIED, AND       *887
C     * CONFIDENCE INTERVALS ARE PRINTED FOR THE INITIAL AND TRANS-      *888
C     * ITIONAL PROBABILITIES.                                           *889
C     *                                                                  *890
C     *     THIS SUBROUTINE IS CALLED ONCE BY THE MAIN PROGRAMME.        *891
C     * MRKOV2() ALSO MAKES ONE CALL TO SUBROUTINE GBS002() FOR THE      *892
C     * INVERSION OF A MATRIX, AND TWO CALLS TO SUBROUTINE CHISQR()      *893
C     * FOR PROBABILITY VALUES.                                          *894
C     *                                                                  *895
C     ********************************************************************896
C                                                                       897
C                                                                       898
C     NBAR  - (REAL) NUMBER OF GRAINS                                   899
C                                                                       900
      DIMENSION NS(6),N1(3),N2(3,3),A(6,9),B(9,6),E(9,6),C(6,6),P(3),   901
     + P1(3),ZR(3,3),MP(3,3),MP1(3,3),MP2(3,3),MP3(3,3),MP4(3,3),       902
     + F(6),FS(6),INN(6,5),ZS(9),P2(3),SUML(3)                          903
      DIMENSION LWK(6),MWK(6)                                           904
      INTEGER Q,Z                                                       905
      REAL MP,MP1,MP2,MP4,INN,MP3,NBAR                                  906
      COMMON/LABEL1/ N2,SUML                                            907
      DO 20 I=1,3                                                       908
      N1(I)=0                                                           909
      DO 18 J=1,3                                                       910
18    N1(I)=N1(I)+N2(I,J)                                               911
C                                                                       912
```

```
C     ALL "SUML" APPEARING IN THIS PROGRAMME                                913
C       HAVE BEEN INSERTED BY K F CHONG.                                    914
C                                                                           915
C     P(I)=(FLOAT(N1(I))         )/FLOAT(NBAR)                            916
      P(I)=(FLOAT(N1(I))+SUML(I))/NBAR                                      917
      DO 20 K=1,3                                                           918
      MP(I,K)=FLOAT(N2(I,K))/FLOAT(N1(I))                                   919
20    CONTINUE                                                              920
      DO 30 I=1,3                                                           921
C     P1(I)=SQRT(FLOAT(N1(I)))                                              922
      P1(I)=SQRT(FLOAT(N1(I))+SUML(I))                                      923
      DO 30 J=1,3                                                           924
30    MP1(I,J)=SQRT(MP(I,J))                                                925
      DO 40 I=1,6                                                           926
      DO 40 J=1,9                                                           927
40    A(I,J)=0                                                              928
      K=-3                                                                  929
      DO 45 II=1,3                                                          930
      K=K+3                                                                 931
      DO 45 JJ=1,3                                                          932
45    A(II,JJ+K)=MP1(II,JJ)                                                 933
      A(4,2)=-P1(2)*P(1)*MP1(1,2)                                           934
      A(4,4)=P1(1)*P(2)*MP1(2,1)                                            935
C                                                                           936
C     A(5,3)=P1(3)*P(1)*MP1(1,3)                                            937
C     *****     MINUS-SIGN (-) ADDED TO A(5, 3)  BY K F CHONG.              938
C     A(5,3)= P1(3)*P(1)*MP1(1,3)                                           939
      A(5,3)=-P1(3)*P(1)*MP1(1,3)                                           940
C                                                                           941
      A(5,7)=P1(1)*P(3)*MP1(3,1)                                            942
      A(6,6)=-P1(3)*P(2)*MP1(2,3)                                           943
      A(6,8)=P1(2)*P(3)*MP1(3,2)                                            944
      F(1)=F(2)=F(3)=0                                                      945
      F(4)=(MP(2,1)*P(2)-MP(1,2)*P(1))*P1(1)*P1(2)                          946
      F(5)=(MP(3,1)*P(3)-MP(1,3)*P(1))*P1(1)*P1(3)                          947
      F(6)=(MP(3,2)*P(3)-MP(2,3)*P(2))*P1(2)*P1(3)                          948
      DO 50 I=1,6                                                           949
      DO 50 J=1,9                                                           950
50    B(J,I)=A(I,J)                                                         951
      DO 55 II=1,6                                                          952
      DO 55 JJ=1,6                                                          953
      C(II,JJ)=0                                                            954
      DO 55 L=1,9                                                           955
      C(II,JJ)=C(II,JJ)+A(II,L)*B(L,JJ)                                     956
55    CONTINUE                                                              957
C                                                                           958
C                                                                           959
C     CALL P0037(C,G,D)                                                     960
C     CALL P1041(C)                                                         961
C                                                                           962
C     C IS THE INVERSE                                                      963
C     EMRLIB SUBROUTINE IS CALLED   (LISTING FOLLOWS AT END)                964
C                                                                           965
      CALL GBS002(C,6,AWK,LWK,MWK)                                          966
      DO 60 I=1,9                                                           967
      DO 60 J=1,6                                                           968
      E(I,J)=0                                                              969
      DO 60 L=1,6                                                           970
      E(I,J)=E(I,J)+B(I,L)*C(L,J)                                           971
60    CONTINUE                                                              972
      DO 70 I=1,9                                                           973
      ZS(I)=0                                                               974
      DO 70 J=1,6                                                           975
      ZS(I)=ZS(I)+E(I,J)*F(J)                                               976
70    CONTINUE                                                              977
      U=0                                                                   978
      DO 80 I=1,6                                                           979
      FS(I)=0                                                               980
      DO 80 J=1,6                                                           981
      FS(I)=FS(I)+F(J)*C(J,I)                                               982
80    CONTINUE                                                              983
      DO 90 I=1,6                                                           984
```

```
90    U=U+FS(I)*F(I)                                                    985
      P2(1)=7.815                                                       986
      P2(2)=11.345                                                      987
      P2(3)=U                                                           988
C                                                                       989
      K=-3                                                              990
      DO 95 II=1,3                                                      991
      K=K+3                                                             992
      DO 95 JJ=1,3                                                      993
      ZR(II,JJ)=ZS(JJ+K)                                                994
      MP2(II,JJ)=MP(II,JJ)/P(JJ)-MP1(II,JJ)/(P1(II)*P(JJ))*ZR(II,JJ)    995
      MP3(II,JJ)=0                                                      996
      MP3(II,II)=P(II)                                                  997
95    CONTINUE                                                          998
      DO 100 I=1,3                                                      999
      DO 100 J=1,3                                                      1000
      MP4(I,J)=0                                                        1001
      DO 100 L=1,3                                                      1002
100   MP4(I,J)=MP4(I,J)+MP2(I,L)*MP3(L,J)                               1003
C                                                                       1004
C     CALL P1041(MP2)                                                   1005
C     CALL P1041(MP4)                                                   1006
C     CALL P1041(P2)                                                    1007
      PRINT 101                                                         1008
101   FORMAT("-", 12X,"MATRIX OF PARAMETERS,  C(I,J)"/)                 1009
      DO 102 I=1,3                                                      1010
102   PRINT 103, (MP2(I,J),J=1,3)                                       1011
103   FORMAT(" ", 2X,3F15.8)                                            1012
      PRINT 104                                                         1013
104   FORMAT("-", 11X,"MATRIX P(I;J) = C(I,J) * LAMBDA(P)"/)            1014
      DO 105 I=1,3                                                      1015
105   PRINT 103, (MP4(I,J),J=1,3)                                       1016
      PRINT 107, P2(3)                                                  1017
107   FORMAT("-", "IDEAL GRANITE  (CHI-SQUARED)",F15.4)                 1018
      PRINT 108                                                         1019
108   FORMAT(" ", "THEORETICAL CHI-SQUARED AT 0.05 IS   3.84")          1020
C                                                                       1021
      DO 110 I=1,3                                                      1022
      DO 110 J=1,3                                                      1023
110   ZR(I,J)=MP2(I,J)-MP2(J,I)                                         1024
C                                                                       1025
C     CALL P1041(ZR)                                                    1026
      PRINT 111                                                         1027
111   FORMAT("1", "VERIFICATION OF SYMMETRY  - FOR C(I,J)")             1028
      PRINT 112, ZR                                                     1029
112   FORMAT(1X, 25X, 3F12.4)                                           1030
      DO 115 II=1,3                                                     1031
      P2(II)=0                                                          1032
      DO 115 JJ=1,3                                                     1033
      P2(II)=P2(II)+MP2(II,JJ)*P(JJ)                                    1034
115   CONTINUE                                                          1035
C                                                                       1036
C     CALL P1041(P2)                                                    1037
      PRINT 116                                                         1038
116   FORMAT("0", "VERIFICATION OF UNIT-VECTOR - C(I,I)*IP(I)")         1039
      PRINT 117, P2                                                     1040
117   FORMAT(1X, 25X,3F12.8)                                            1041
C                                                                       1042
      DO 120 I=1,3                                                      1043
      NS(I)=N1(I)+INT(SUML(I))                                          1044
      NS(I+3)=N2(I,I)                                                   1045
120   CONTINUE                                                          1046
C                                                                       1047
C -------------------------------------------------------------------   1048
C     CALCULATION OF CONFIDENCE INTERVALS                               1049
C                                                                       1050
      DO 125 Q=1,6                                                      1051
```

```
      DO 125 K=2,5                                                      1052
125   INN(Q,K)=0.0                                                      1053
      DO 127 I=1,3                                                      1054
      INN(I,1)=P(I)                                                     1055
127   INN(I+3,1)=MP(I,I)                                                1056
C                                                                       1057
      DO 300 Q=1,6                                                      1058
C                                                                       1059
      L=2*NS(Q)                                                         1060
      CALL CHISQR(L,CHI5,CHI95,CHI1,CHI99)                              1061
      IF(Q .GT. 3) GO TO 200                                            1062
      R=2*NBAR-NS(Q)+1                                                  1063
      GO TO 210                                                         1064
200   R=2*NS(Q-3)-NS(Q)+1                                               1065
210   INN(Q,3)=CHI5/(CHI5/2.+R)                                         1066
      INN(Q,5)=CHI1/(CHI1/2.+R)                                         1067
C                                                                       1068
      L=L+2                                                             1069
      CALL CHISQR(L,CHI5,CHI95,CHI1,CHI99)                              1070
      IF(Q .GT. 3) GO TO 220                                            1071
      R=2*NBAR-NS(Q)                                                    1072
      GO TO 230                                                         1073
220   R=2*NS(Q-3)-NS(Q)                                                 1074
230   INN(Q,2)=CHI95/(CHI95/2.+R)                                       1075
      INN(Q,4)=CHI99/(CHI99/2.+R)                                       1076
C                                                                       1077
300   CONTINUE                                                          1078
C                                                                       1079
C                                                                       1080
C  - - - - - - - - - - - - - - - - -  PRINT THE RESULTS                 1081
C                                                                       1082
C     CALL P1041(INN)                                                   1083
      PRINT 342                                                         1084
342   FORMAT("-", " CONFIDENCE INTERVALS FOR IP(I) AND TP(I,I)"/        1085
     + "0", 17X,"LEVEL=0.05",8X,"LEVEL=0.01"/                           1086
     + " ", 6X,"IP(I)",2("    UPPER    LOWER"))                        1087
      DO 350 I=1,3                                                      1088
      PRINT 344, (INN(I,J),J=1,5)                                       1089
344   FORMAT(1X, 2X,5F9.4)                                              1090
350   CONTINUE                                                          1091
      PRINT 346                                                         1092
346   FORMAT("0", 4X,"TP(I,I)",2("    UPPER    LOWER"))                1093
      DO 360 I=4,6                                                      1094
360   PRINT 344, (INN(I,J),J=1,5)                                       1095
C                                                                       1096
      DELTAC=0.                                                         1097
      DO 370 J=1,3                                                      1098
      DO 370 I=1,3                                                      1099
      DELTAC=DELTAC + (MP2(I,J)-1.)**2                                  1100
370   CONTINUE                                                          1101
      PRINT 380, DELTAC                                                 1102
380   FORMAT("-", "THE VALUE FOR DELTA-C IS",F8.4)                      1103
C                                                                       1104
      RETURN                                                            1105
      END                                                               1106

      SUBROUTINE CHISQR(DF,CHI5,CHI95,CHI1,CHI99)                       1107
C                                                                       1108
C     ********************************************************************* 1109
C     *                                                               *   1110
C     *     SUBROUTINE CHISQR() CALCULATES 5 PERCENT, 95 PERCENT,     *   1111
C     * 1 PERCENT, AND 99 PERCENT CHI-SQUARE PROBABILITY VALUES FOR   *   1112
C     * A GIVEN NUMBER OF DEGREES OF FREEDOM.                         *   1113
C     *                                                               *   1114
C     *     THIS SUBROUTINE IS CALLED AS MANY AS FOUR TIMES BY SUB-   *   1115
C     * ROUTINE MRKOV(), AND CALLED TWICE BY SUBROUTINE MRKOV2().     *   1116
C     *                                                               *   1117
C     ********************************************************************* 1118
C                                                                       1119
```

```
C     LIST OF PARAMETERS:                                                    1120
C                                                                            1121
C          INPUT:                                                            1122
C            DF    - (INTEGER) NUMBER OF DEGREES OF FREEDOM.                 1123
C                                                                            1124
C          OUTPUT:                                                           1125
C            CHI5  - (REAL) 5 PERCENT CHI-SQUARE PROBABILITY VALUE.          1126
C            CHI95 - (REAL) 95 PERCENT CHI-SQUARE PROBABILITY VALUE.         1127
C            CHI1  - (REAL) 1 PERCENT CHI-SQUARE PROBABILITY VALUE.          1128
C            CHI99 - (REAL) 99 PERCENT CHI-SQUARE PROBABILITY VALUE.         1129
C                                                                            1130
      INTEGER DF,DUMMY1(3,3)                                                 1131
      REAL DUMMY2(3),BBARR(100),ABARR(100),CBARR(100),DBARR(100)             1132
      COMMON/LABEL1/ DUMMY1,DUMMY2,BBARR,ABARR,CBARR,DBARR                   1133
C                                                                            1134
      IF(DF .GT. 100) GO TO 10                                               1135
      CHI5=ABARR(DF)                                                         1136
      CHI95=BBARR(DF)                                                        1137
      CHI1=CBARR(DF)                                                         1138
      CHI99=DBARR(DF)                                                        1139
      GO TO 9999                                                             1140
C                                                                            1141
10    ZP1=-1.645                                                             1142
      ZP2= 1.645                                                             1143
      ZP3=-2.326                                                             1144
      ZP4= 2.326                                                             1145
      FLDF=FLOAT(DF)                                                         1146
      CHI5= FLDF * (1.-(2./9./FLDF)+ZP1*SQRT(2./9./FLDF))**3                 1147
      CHI95=FLDF * (1.-(2./9./FLDF)+ZP2*SQRT(2./9./FLDF))**3                 1148
      CHI1= FLDF * (1.-(2./9./FLDF)+ZP3*SQRT(2./9./FLDF))**3                 1149
      CHI99=FLDF * (1.-(2./9./FLDF)+ZP4*SQRT(2./9./FLDF))**3                 1150
C                                                                            1151
9999  RETURN                                                                 1152
      END                                                                    1153

      SUBROUTINE GBS002(A,N,D,L,M)                                           1154
C                                                                            1155
C     ************************************************************** 1156
C     *                                                            * 1157
C     *     SUBROUTINE GBS002() PERFORMS MATRIX INVERSION, USING   * 1158
C     * THE STANDARD GAUSS-JORDAN TECHNIQUE.                       * 1159
C     *                                                            * 1160
C     *     THIS SUBROUTINE IS CALLED ONCE BY SUBROUTINE MRKOV2(). * 1161
C     *                                                            * 1162
C     ************************************************************** 1163
C                                                                            1164
      DIMENSION A(1),L(1),M(1)                                               1165
C                                                                            1166
      D=1.0                                                                  1167
      NK=-N                                                                  1168
      DO 80 K=1,N                                                            1169
      NK=NK+N                                                                1170
      L(K)=K                                                                 1171
      M(K)=K                                                                 1172
      KK=NK+K                                                                1173
      BIGA=A(KK)                                                             1174
      DO 20 J=K,N                                                            1175
      IZ=N*(J-1)                                                             1176
      DO 20 I=K,N                                                            1177
      IJ=IZ+I                                                                1178
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20                                    1179
   15 BIGA=A(IJ)                                                             1180
      L(K)=I                                                                 1181
      M(K)=J                                                                 1182
   20 CONTINUE                                                               1183
      J=L(K)                                                                 1184
      IF(J-K) 35,35,25                                                       1185
```

```
25 KI=K-N                                                          1186
   DO 30 I=1,N                                                     1187
   KI=KI+N                                                         1188
   HOLD=-A(KI)                                                     1189
   JI=KI-K+J                                                       1190
   A(KI)=A(JI)                                                     1191
30 A(JI) =HOLD                                                     1192
35 I=M(K)                                                          1193
   IF(I-K) 45,45,38                                                1194
38 JP=N*(I-1)                                                      1195
   DO 40 J=1,N                                                     1196
   JK=NK+J                                                         1197
   JI=JP+J                                                         1198
   HOLD=-A(JK)                                                     1199
   A(JK)=A(JI)                                                     1200
40 A(JI) =HOLD                                                     1201
45 IF(BIGA) 48,46,48                                               1202
46 D=0.0                                                           1203
   RETURN                                                          1204
48 DO 55 I=1,N                                                     1205
   IF(I-K) 50,55,50                                                1206
50 IK=NK+I                                                         1207
   A(IK)=A(IK)/(-BIGA)                                             1208
55 CONTINUE                                                        1209
   DO 65 I=1,N                                                     1210
   IK=NK+I                                                         1211
   IJ=I-N                                                          1212
   DO 65 J=1,N                                                     1213
   IJ=IJ+N                                                         1214
   IF(I-K) 60,65,60                                                1215
60 IF(J-K) 62,65,62                                                1216
62 KJ=IJ-I+K                                                       1217
   A(IJ)=A(IK)*A(KJ)+A(IJ)                                         1218
65 CONTINUE                                                        1219
   KJ=K-N                                                          1220
   DO 75 J=1,N                                                     1221
   KJ=KJ+N                                                         1222
   IF(J-K) 70,75,70                                                1223
70 A(KJ)=A(KJ)/BIGA                                                1224
75 CONTINUE                                                        1225
   D=D*BIGA                                                        1226
   A(KK)=1.0/BIGA                                                  1227
80 CONTINUE                                                        1228
   K=N                                                             1229
100 K=(K-1)                                                        1230
   IF(K) 150,150,105                                               1231
105 I=L(K)                                                         1232
   IF(I-K) 120,120,108                                             1233
108 JQ=N*(K-1)                                                     1234
   JR=N*(I-1)                                                      1235
   DO 110 J=1,N                                                    1236
   JK=JQ+J                                                         1237
   HOLD=A(JK)                                                      1238
   JI=JR+J                                                         1239
   A(JK)=-A(JI)                                                    1240
110 A(JI) =HOLD                                                    1241
120 J=M(K)                                                         1242
   IF(J-K) 100,100,125                                             1243
125 KI=K-N                                                         1244
   DO 130 I=1,N                                                    1245
   KI=KI+N                                                         1246
   HOLD=A(KI)                                                      1247
   JI=KI-K+J                                                       1248
   A(KI)=-A(JI)                                                    1249
130 A(JI) =HOLD                                                    1250
   GO TO 100                                                       1251
150 RETURN                                                         1252
   END                                                             1253
```

APPENDIX 2

Method of counting transitions

This appendix consists of two parts. The first part is a summary prepared by M.E. Demina on the method used for counting the transitions between grains in the Meech Lake aplite. This is followed by examples of actual sequences of transitions counted. Another example was shown previously in Table 2A of the text. These examples can be used for testing the computer program in Appendix 1.

A thin section of size 60 x 35 mm was prepared for each sample. Each thin section was placed on a special device (MYI-1) similar to the Shand micrometer stage, at the Laboratory of Mathematical Geology in Leningrad. MYI-1 permits the movement of a thin section along a line parallel to the horizontal crosshair of the microscope. Initially each thin section was placed in such a position that the first grain on the right side near the upper border coincided with the intersection of the horizontal and vertical crosshairs of the microscope. Then the thin section was moved step by step to the left until the last grain of a traverse. During this process, the compositions of all grains traversed were determined by writing 1 for K-feldspar, 2 for quartz, and 3 for plagioclase. The resulting sequence (e.g. ...,2,1,2,2,3,1,...) is not continuous but may be interrupted by grains of other minerals which were ignored. A semicolon indicates whenever the sequence was interrupted by grains of other minerals, and at the end of each traverse. Semicolons were also entered for granophyric textures, mirmekites, zones of cataclasis, veinlets and other nontypical features. Twinned plagioclase were accepted as single grains and perthites were ignored in that grains with perthites were accepted as grains of K-feldspar.

If a plagioclase grain was surrounded by a narrow rim composed of albite, this rim was ignored and the grain was accepted as a single plagioclase grain. If a grain of quartz was crushed and consisted of fragments which had not been moved with respect to one another, while the primary boundaries of the grain were preserved, this grain was accepted as a single quartz grain.

The length of each subsequence enclosed by two semicolons is at least 10 grains because shorter subsequences were ignored completely. In total, about 1000 transitions were counted for each thin section along parallel traverses. The distance between traverses was kept to more than three times the average grain diameter and care was taken not to include the same grain in more than a single traverse.

The preceding method of counting transitions was developed by Aradadsheva-Tzvetjova et al. (1972). The K-feldspars in our sections were stained by using the method of Kigaiy (1960).

Examples of sequences of transitions between grains

Sample 3 (First order Markov chain; ideal granite)

12121121211221222121212121211312112121211212121312212211 2
12122131121212123123213212121111212212;213121223112121
13212221212121212312212121222111212121213212121211213
21123211221212131213121112131311131212121212121;21211211
21212121111212111212212122232131221212212312131132 1;1213
12321312111211121131131222312312131211121222112221221 3
12121121212121212221121312112121222122123221112211311112
13121121131211211211112121312121212121132121221213213 2
12121212212121212;12121212121321123121212121212121312212
12211212121212121212312121212211112222113322 1;21211211
21121131322112122112131122121211311122311212 2;12123212
12132121321211112322221212121231212 1;21221212121221213
11321121221131211113111313111113212131213212121121311 2
22211113112 1;12121213211212132121313131212212131121212
21212122113232131122121213121312222131212121213111132 13
12121222121312212121213322211313112 13;21212121122121 21
22213121211212112221212122121312121122112222122112213121
21212121 2;12122213112131212121321211131212213131211212
31131212 12;12131222132212121122123112;11211112121121 32
31312121311131312212121121131222121211131211121122121212
12112312113212131211212121211213121312121 2;

Appendix 2 (cont.)

Sample 8 (First order Markov chain; non-ideal granite)

112121212111212231212121121222122311121222122121211212121
132111111113121121111;112112112121123212121213121112
112221213211211122112111121121131121211;321121121212112
212311;121221312121111211223111;121211121212212121121112
121211112121121112312312111131211232121;112212131312212
131211231212123111113112121112123212121;321211112212121
121212111121112112131322121121113111;112231211211131121
212311211111121131311111312;111212121211212122321213312
211132;113111121222112232;113111311111121211112121321212
11;122122211321121211211231211131111121111212132121212
21;321213212212212121212132;212111312111121123121212121212
111223121311212111112;132121112312112121121121121112;121112
131111;122121312112121211212121222121112121311132121112
13111211;212131211221222121;121211111123121111211211211111
212111122221212121111121111;113111121211112213131211121
21212121;111211222211122;111113123111212121121212111212221
212121212121121;111123121211213122;112112121111212121211
121111231311;312121111211121123131112;313111221211112112
1211112;213121111213121111;121113212112112123212122131212
122113112121311313211211212121212121212121121212121222121
122212112121212;

Sample 28 (Second order Markov chain)

212122232122131121223123132131212231331321321312313312
213222132321221321232323331232122131322332322221213233
131232133312231313121113;321212121212132311213213132313
213123311333212121323313232222312213122233231323221213
121312212323131323232222322222331231313133;321331213312 2
222312212231233223212121233123122313232122231233131233
322313121212312312321312312233232132231312231312232131
213213231321223213131312222132321313321323131312121121
323121223121331232213321223233322232213231121312121213
112121321;122322223233131313223133322223323313323122131
332121212131321311312231213122132322323321213312212 1
221322132231212121233122133112312232222331222222333131
331321312132332333121222312131321232232123232233232131
211321222131222121311321132311221211;212121321313131 31
222131222233332231313312313332323122223322313132133131
232233121312113212123132233131222122312233221313221213
231332231321321313121222121231311313121211323331313221
212121223221212121323121 2;323231222323122123323132323 1
332222313233231213223221313222313222322213231212232221
322232313212233133213133233213231112213222212322231232
313231313313231212332313122313312131322123232233232321
312321121133322321;

APPENDIX 3

Appendix 3-1: Matrices of transition frequencies (in numbers of grains) for the sequences of grains which are similar to chains of the first order and have a matrix of transition probabilities similar to that of ideal granite. A-stock; B-dykes.

A

	2			4			5			6			7			8			9	
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	206	309	75	194	329	75	56	244	128	83	221	135	63	244	173	266	290	57	165	249	135
2	307	74	31	333	74	13	238	125	88	227	84	93	260	97	67	282	47	22	254	103	44
3	74	32	2	70	18	2	130	88	19	124	101	16	158	82	15	63	22	1	131	48	6
4	74	110	20	66	109	17	9	32	13	15	37	31	11	32	19	119	113	21	52	72	40
12	226	57	18	258	53	11	131	67	43	131	46	41	148	59	35	229	37	16	156	64	26
13	53	21	1	60	12	1	76	47	6	72	57	5	104	59	50	43	12	1	103	27	5
21	105	158	40	104	181	43	30	137	69	39	115	72	34	132	90	110	141	25	72	121	57
22	57	10	6	59	13	0	67	35	22	46	14	23	61	23	13	32	8	6	61	29	12
23	19	11	1	8	4	1	44	32	12	44	37	11	44	18	4	16	6	0	23	19	1
31	24	35	15	21	34	13	14	72	43	27	66	31	18	78	61	25	29	9	40	53	37
32	21	4	7	12	5	1	40	23	23	47	23	29	49	15	18	18	1	0	32	9	6
33	2	0	0	1	1	0	10	8	1	8	7	0	9	5	1	1	0	0	5	1	0
f_t	590	412	108	598	420	90	426	451	237	439	404	241	480	424	255	612	351	86	549	401	185
f_t^p	596	421	108	607	429	92	429	457	237	440	410	244	486	426	256	631	363	86	553	406	186

	12			13			14			15			19			20			21	
1	108	245	125	189	303	94	144	304	138	206	228	109	305	303	73	29	138	133	233	244	117
2	219	85	72	304	85	46	312	42	31	229	50	44	310	17	17	152	157	139	255	46	38
3	119	80	12	93	49	4	120	39	1	102	51	5	65	25	0	120	152	80	105	49	4
11	20	53	32	67	95	26	42	77	23	86	75	41	130	139	27	4	11	14	89	93	51
12	131	45	34	212	56	31	251	27	26	161	34	27	269	15	14	53	49	36	180	34	28
13	73	46	6	67	24	3	98	30	0	76	28	3	53	20	0	48	62	23	77	36	3
21	56	102	60	95	157	50	72	159	79	77	104	44	147	131	30	16	71	64	107	95	48
22	42	23	20	62	16	7	29	10	3	37	5	6	15	2	0	22	48	51	38	1	6
23	36	29	5	24	21	1	20	9	1	25	17	2	12	3	0	52	52	34	26	11	1
31	31	55	31	26	50	18	30	65	24	34	44	21	22	27	14	9	55	55	35	52	17
32	44	16	17	28	13	8	31	5	2	29	9	11	21	0	3	38	59	51	35	11	3
33	7	5	0	2	2	0	1	0	0	1	4	0	0	0	0	20	37	22	2	2	0
f_t	448	376	211	586	435	148	576	385	160	543	323	158	681	344	90	300	448	352	594	339	158
f_t^p	454	384	213	590	439	148	581	386	161	554	333	160	694	350	90	302	452	354	600	342	159

	22			23			24			26			27			32			33	
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	352	261	58	283	308	87	154	282	117	146	242	82	186	274	73	248	329	76	86	197	136
2	257	32	23	308	30	14	298	64	42	229	133	73	275	127	47	335	77	16	190	72	118
3	55	26	2	86	17	0	104	53	3	93	62	14	73	46	4	70	24	2	141	112	44
11	183	134	26	120	125	37	50	71	32	44	77	30	78	80	25	89	120	29	22	41	23
12	213	21	19	269	24	12	209	42	31	132	65	42	174	73	23	258	54	11	112	33	50
13	37	18	2	71	16	0	77	38	1	43	30	7	40	30	3	55	20	1	66	45	23
21	120	107	25	126	145	34	70	155	71	70	118	38	87	131	35	127	163	40	32	92	63
22	26	5	1	26	4	0	44	12	8	67	45	18	72	40	15	51	22	4	28	21	23
23	15	8	0	13	1	0	25	14	2	43	22	6	32	13	1	12	3	1	53	47	16
31	30	17	5	37	34	14	33	55	13	31	50	13	19	42	12	24	39	5	31	61	49
32	18	4	3	13	2	2	41	9	3	27	21	14	24	13	8	22	1	1	48	18	45
33	1	0	0	0	0	0	2	1	0	6	7	1	1	3	0	1	1	0	21	18	5
f_t	671	312	83	678	352	103	553	404	160	470	437	171	533	449	123	653	428	96	419	380	297
f_t^p	688	321	85	683	355	103	559	404	162	475	443	175	538	454	124	671	434	96	422	383	301

	34			35			38			39			40			41			42	
1	165	333	74	76	223	136	230	319	84	191	290	103	75	345	72	271	336	59	348	262	86
2	330	65	34	215	89	124	316	46	20	289	55	37	349	149	27	334	43	14	265	16	18
3	73	35	1	145	113	22	85	18	2	108	32	1	67	32	3	57	17	0	83	22	0
11	58	88	17	14	37	25	74	119	35	52	101	34	10	53	12	110	131	28	181	117	47
12	263	43	22	118	45	59	262	38	16	220	38	31	238	90	15	283	39	12	235	11	15
13	52	21	1	71	57	8	72	10	2	79	22	1	45	25	2	46	13	0	69	17	0
21	87	191	50	35	115	65	127	147	41	100	134	52	53	242	49	135	175	23	125	107	30
22	41	15	8	41	18	30	39	5	2	43	7	5	93	49	6	36	3	2	10	3	2
23	20	14	0	67	45	10	12	7	0	28	9	0	19	7	1	10	4	0	13	5	0
31	17	49	6	26	71	46	28	50	7	38	52	17	12	45	10	23	26	7	39	35	8
32	25	6	4	55	24	33	13	2	2	23	8	1	18	8	6	15	1	0	19	2	1
33	1	0	0	7	11	4	1	1	0	1	0	0	3	0	0	0	0	0	0	0	0
f_t	673	429	109	435	428	280	633	382	105	584	381	141	492	525	102	666	391	74	696	209	105
f_t^p	577	435	109	437	430	282	636	386	106	592	382	142	497	528	102	670	396	74	703	301	105

Appendix 3-1 (cont.)

	43			46			48			50			51			53			54		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	125	242	116	170	108	63	233	280	64	214	307	47	174	270	83	176	294	101	145	315	109
2	242	66	79	319	100	34	285	113	18	316	148	24	279	107	60	287	68	44	320	82	15
3	118	78	19	56	42	3	56	24	3	36	34	2	73	71	12	104	40	4	91	12	12
11	29	63	32	55	92	20	110	119	24	76	121	17	49	103	22	49	103	22	42	78	24
12	157	36	47	226	62	19	196	72	9	210	88	9	182	57	28	204	54	32	240	62	11
13	61	46	8	35	26	2	41	18	3	26	20	1	36	41	6	69	27	4	88	19	2
21	63	126	48	97	181	35	116	136	32	120	168	25	99	134	46	95	141	51	80	173	64
22	36	16	14	65	27	8	71	35	6	88	45	12	57	28	19	55	6	7	63	16	2
23	47	23	9	20	13	1	13	5	0	9	13	1	31	24	5	32	12	0	12	3	0
31	30	52	35	18	31	7	26	23	7	16	15	5	26	32	14	28	49	26	22	60	19
32	48	13	17	24	10	7	15	5	3	16	15	3	40	19	12	27	8	5	15	4	2
33	9	8	2	0	3	0	2	1	0	1	1	0	6	5	1	3	1	0	2	0	0
f	483	387	215	541	453	101	597	416	83	568	488	72	527	446	156	571	399	148	569	417	156
f_t^p	488	389	216	550	455	101	598	421	85	571	491	73	528	452	156	574	403	149	574	421	126

	55			56			57			59			60			61			62		
1	175	272	112	253	289	70	261	283	69	145	279	103	107	232	124	169	325	92	164	270	87
2	272	59	29	294	56	33	276	69	32	275	58	53	231	108	84	335	59	33	270	111	46
3	109	32	4	66	38	6	77	24	3	107	49	6	127	82	16	83	41	0	89	44	1
11	42	78	24	64	84	25	99	123	27	116	117	26	26	72	33	23	51	31	50	94	23
12	240	62	11	200	45	23	221	45	19	211	47	21	202	39	37	123	62	44	171	73	24
13	93	27	3	44	22	4	52	15	2	69	31	3	76	37	10	65	26	0	58	29	0
21	84	135	53	119	142	32	112	128	34	79	152	41	58	111	61	90	191	52	81	147	41
22	63	16	2	45	10	4	45	5	6	45	17	7	43	9	6	59	27	21	68	25	17
23	23	6	0	17	14	1	22	9	1	34	16	3	41	38	5	18	15	0	31	14	1
31	25	50	32	32	23	11	30	35	9	30	51	25	26	60	31	29	37	17	33	40	15
32	25	4	2	25	6	7	15	4	4	29	10	10	46	19	16	33	5	2	27	12	5
33	3	0	1	4	1	1	3	0	0	4	2	0	9	6	0	0	0	0	0	1	0
f	559	360	145	612	383	110	613	377	104	527	386	162	463	423	225	586	427	124	521	427	134
f_t^p	563	365	145	617	387	111	620	382	104	535	387	162	467	428	226	590	430	125	524	430	134

	65			66			67			68			70			71			72		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	227	282	70	119	338	68	201	333	73	162	218	95	254	304	80	196	274	87	163	272	94
2	275	72	36	333	97	32	323	39	14	313	78	19	311	32	8	275	98	27	263	121	26
3	77	28	0	75	25	2	74	12	1	98	16	3	73	15	1	87	28	2	102	18	0
11	76	124	24	25	72	20	71	98	29	42	99	20	104	120	26	65	98	30	4	88	29
12	202	54	26	247	64	26	280	31	11	239	63	13	269	29	6	180	73	19	173	77	21
13	50	19	0	50	17	1	61	10	1	77	15	2	64	15	1	62	24	1	79	15	0
21	116	122	33	70	219	39	97	189	34	90	166	55	121	146	41	102	133	38	79	132	49
22	53	10	8	65	27	5	29	5	3	64	10	4	27	3	2	71	18	8	74	42	4
23	25	9	0	23	8	1	12	2	0	17	1	1	8	0	0	23	3	1	23	3	0
31	32	32	11	23	42	9	30	32	10	29	49	20	27	33	12	27	42	16	34	52	15
32	19	7	2	19	5	1	11	1	0	9	5	2	14	0	0	22	6	0	15	2	1
33	0	0	0	2	0	0	1	0	0	3	0	0	1	0	0	1	1	0	0	0	0
f	579	383	105	525	462	102	597	376	87	575	410	117	638	351	89	557	400	117	529	410	120
f_t^p	587	384	108	533	403	102	605	379	88	578	413	118	646	352	89	564	403	117	433	412	120

	74			75			76			77			78			79			80		
1	215	273	87	94	260	128	222	294	51	184	323	48	189	289	94	85	326	88	185	291	53
2	267	89	16	261	86	46	290	134	18	325	108	9	289	31	15	326	90	20	287	135	14
3	91	12	3	127	47	8	51	17	11	43	13	0	96	13	0	87	21	4	56	11	0
11	75	103	34	22	49	22	77	125	19	47	103	14	73	86	29	20	54	11	74	105	16
12	192	67	12	171	55	33	194	85	12	238	73	7	252	25	10	246	63	14	194	85	10
13	74	11	2	88	37	3	40	11	0	37	10	0	82	12	0	66	19	2	45	8	0
21	104	128	34	52	142	64	125	137	65	104	190	28	81	155	48	52	206	63	104	149	33
22	66	21	2	57	23	5	80	47	6	77	29	2	22	5	4	63	22	5	84	46	3
23	14	1	1	35	7	4	11	3	1	6	3	0	14	1	0	18	1	1	11	3	-
31	35	38	16	20	65	41	15	30	6	11	26	6	34	44	17	13	60	13	19	33	4
32	9	1	2	31	8	8	15	2	0	8	5	0	11	1	1	15	4	1	7	4	0
33	3	0	0	43	1	0	1	0	0	0	0	0	0	0	2	1	1	0	0	0	
f	575	372	106	482	393	182	567	442	69	535	442	56	572	335	109	499	436	112	519	436	07
f_t^p	591	384	1-7	498	305	192	570	446	70	538	447	56	579	337	109	505	440	113	533	440	77

Appendix 3-1 (cont.)

	81			82			84			85			86			88			89		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	115	348	53	241	277	61	101	350	31	69	328	72	105	142	158	204	293	62	122	200	120
2	345	103	6	279	80	17	347	154	16	337	93	29	155	149	88	299	114	88	299	114	8
3	54	5	0	58	20	1	32	15	0	62	38	1	140	104	16	36	14	0	188	109	89
11	24	80	10	90	118	29	23	71	7	6	50	10	30	36	37	87	86	29	42	55	31
12	266	74	5	201	62	12	233	103	11	236	69	23	59	54	21	208	79	5	98	58	51
13	49	4	0	46	14	1	22	9	0	45	27	0	90	57	8	52	10	0	63	36	20
21	75	231	36	118	131	25	69	251	21	57	230	48	34	51	64	102	168	26	48	93	45
22	78	25	0	63	12	4	103	48	3	70	16	6	53	51	35	80	30	3	47	35	23
23	5	1	0	11	6	0	10	6	0	16	11	1	41	35	6	4	3	0	49	30	9
31	16	30	7	27	24	7	9	20	3	6	43	13	38	49	50	15	32	7	42	48	41
32	1	3	1	13	6	1	10	3	2	29	8	0	39	30	28	9	5	0	51	15	24
33	0	0	0	1	0	0	0	0	0	1	0	0	6	9	1	0	0	0	17	12	3
f_t	516	454	59	579	376	79	482	517	47	569	459	101	405	383	260	559	421	70	453	386	247
f_t^p	521	457	59	588	379	79	488	520	47	474	461	102	416	399	269	566	423	71	462	394	250

	90			91			92			93			94			95			97		
1	81	203	145	195	254	90	259	301	64	313	194	89	101	265	108	179	196	64	257	296	76
2	202	158	56	254	55	49	298	35	23	196	41	43	265	90	92	293	94	36	295	40	14
3	133	81	33	145	56	14	86	56	5	86	49	6	108	99	12	67	33	4	80	11	2
11	20	33	25	66	86	37	103	129	23	143	94	52	28	53	20	57	96	21	91	126	34
12	98	77	25	177	36	29	251	28	21	132	26	30	165	49	49	200	70	22	245	38	12
13	102	34	9	51	37	2	53	11	0	55	31	3	55	46	7	40	20	3	66	8	2
21	36	101	63	94	118	36	118	141	34	95	62	25	53	148	61	92	158	36	132	128	31
22	472	61	24	30	8	12	29	5	1	29	4	4	48	21	19	66	17	10	36	2	2
23	34	17	5	30	15	3	17	6	0	24	17	1	51	38	3	23	10	1	11	3	0
31	23	63	55	31	41	13	36	27	7	49	27	6	20	63	25	23	39	4	32	38	9
32	32	18	6	38	10	7	15	1	1	30	9	9	50	18	24	23	4	3	9	0	0
33	9	5	0	3	2	0	0	0	0	5	0	1	2	8	2	2	2	0	2	0	0
f_t	429	416	215	539	358	147	624	356	87	596	280	141	474	447	214	539	423	104	629	349	93
f_t^p	438	420	215	552	376	148	633	357	87	638	291	142	477	453	214	553	431	107	640	352	93

	98			99			100			101			102			103			104		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	215	302	85	103	202	149	186	306	94	172	252	115	102	255	134	173	296	85	286	325	55
2	301	45	28	218	125	61	315	58	33	264	73	47	261	97	75	292	86	25	324	38	15
3	84	29	0	135	78	16	83	44	1	103	59	11	123	84	20	88	25	6	50	19	0
11	74	108	30	23	46	34	44	102	38	51	82	37	16	57	28	56	95	20	110	150	23
12	236	41	20	116	57	27	237	43	24	180	45	26	149	62	41	216	59	17	271	34	14
13	59	26	0	90	51	8	59	34	1	65	43	7	74	45	14	60	20	5	41	13	0
21	106	150	40	48	99	68	115	153	45	88	119	55	61	125	74	87	157	45	150	146	26
22	39	1	5	61	41	20	46	10	2	49	13	11	55	17	24	60	17	7	35	2	1
23	25	3	0	37	18	6	23	10	0	34	11	2	41	29	4	19	5	1	9	6	0
31	32	39	12	32	54	46	27	45	10	32	46	23	23	70	30	27	41	18	23	22	4
32	23	3	3	40	26	12	31	5	7	34	15	10	56	18	10	14	9	1	15	2	0
33	0	0	0	8	6	2	1	0	0	4	5	2	8	10	2	6	0	0	0	0	0
f_t	602	374	113	454	404	229	386	406	128	537	384	173	491	433	227	554	403	119	666	377	69
f_t^p	611	370	113	660	409	229	591	409	128	545	385	173	493	437	229	561	410	119	672	385	70

	105			106			107			108			109			110			111		
1	97	234	123	168	298	99	128	282	96	151	297	86	139	381	58	76	238	160	392	24	61
2	239	147	70	296	71	39	288	113	56	307	127	21	385	62	18	250	127	38	229	62	26
3	117	77	9	102	39	5	89	64	7	78	30	5	54	20	0	151	48	8	66	21	1
11	18	47	30	53	86	26	36	66	22	48	74	26	36	97	7	12	40	24	219	128	39
12	122	79	31	215	52	27	169	79	31	202	78	16	310	54	14	145	73	19	170	42	21
13	64	54	5	67	29	3	52	40	4	55	26	5	41	16	0	118	37	5	45	15	1
21	50	127	60	85	101	48	65	161	60	83	178	41	89	248	45	42	121	85	124	86	11
22	69	45	31	50	14	7	72	21	19	85	39	3	54	6	2	75	40	12	43	12	4
23	46	20	4	28	9	2	33	20	3	18	3	0	13	4	0	28	7	3	20	5	0
31	26	59	31	29	51	22	25	50	14	19	42	16	13	35	5	22	76	30	34	24	8
32	46	23	8	28	5	5	47	12	5	19	9	2	17	1	1	29	13	5	13	7	1
33	6	3	0	4	1	0	4	3	0	3	1	0	0	0	0	5	3	0	0	1	0
f_t	454	456	203	565	406	146	506	437	160	534	455	113	578	465	74	474	415	207	696	317	88
f_t^p	459	460	203	570	411	146	512	461	160	543	456	114	582	569	76	479	417	207	110	330	89

Appendix 3-1 (cont.)

	113			114			116			117		
	1	2	3	1	2	3	1	2	3	1	2	3
1	265	285	91	124	147	203	27	65	256	59	129	213
2	229	62	26	279	68	21	152	58	54	62	52	62
3	96	17	0	195	58	64	258	59	213	203	64	108
11	106	118	37	39	33	51	2	5	49	10	16	33
12	220	51	12	88	32	25	26	15	24	63	41	22
13	78	13	0	124	37	37	126	23	103	124	27	59
21	112	121	44	43	51	57	8	15	38	17	38	78
22	48	14	6	33	13	10	15	18	18	44	35	18
23	17	4	0	31	11	12	33	6	23	24	16	14
31	43	43	10	42	59	93	17	44	195	31	72	98
32	11	2	3	27	12	18	19	19	20	27	21	16
33	0	0	0	40	10	13	95	30	84	53	19	34
f_t	641	368	113	474	264	317	348	177	530	401	290	375
f_t^P	647	377	113	477	169	323	352	179	539	405	295	382

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	63			64			118			119			120			122			124		
1	254	296	52	182	317	67	250	180	181	237	243	122	158	276	115	225	253	83	197	283	96
2	294	71	21	321	119	12	178	29	31	249	63	28	288	97	38	260	112	33	287	66	19
3	54	19	1	63	16	0	184	27	15	114	36	8	101	49	4	78	39	8	93	21	2
11	97	132	22	52	110	20	105	76	65	96	92	44	47	75	35	89	100	34	60	106	30
12	222	54	17	221	85	9	132	17	22	173	44	24	185	63	25	159	70	22	216	52	14
13	38	13	1	55	12	0	147	20	11	85	31	6	73	57	4	53	25	5	76	18	2
21	130	137	24	109	172	36	67	53	56	94	98	55	84	141	62	102	123	32	105	135	45
22	58	11	1	87	28	3	19	7	3	51	10	1	68	20	9	75	28	8	56	7	2
23	15	6	0	8	4	0	23	3	3	22	4	2	24	12	0	21	10	2	16	3	0
31	27	21	6	21	31	10	75	45	57	43	50	20	25	60	16	33	29	16	32	42	20
32	11	5	2	11	3	0	16	5	6	24	7	3	33	14	2	24	12	3	12	6	2
33	1	0	0	0	0	0	12	2	1	7	1	0	4	0	0	3	4	1	2	0	0
f_t	602	386	74	566	452	79	611	238	226	602	340	158	549	421	154	561	405	125	576	372	148
f_t^P	608	391	74	571	455	79	624	241	229	610	345	158	551	424	155	566	408	125	580	375	118

	125			126			127			128			129			130			131		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	140	313	82	230	323	53	229	279	67	270	274	82	158	289	101	168	273	87	278	256	67
2	310	83	24	325	60	19	272	112	21	267	63	23	282	86	31	268	109	45	255	113	17
3	82	24	2	52	21	0	74	14	1	87	19	4	104	28	2	89	43	4	65	17	0
11	60	106	30	40	81	19	73	135	20	107	119	42	44	80	34	49	95	23	116	124	36
12	216	52	14	224	67	18	258	48	15	202	53	17	196	64	25	165	68	34	174	67	13
13	76	18	2	61	19	2	36	17	0	66	14	2	79	20	2	55	30	2	52	13	0
21	80	178	51	131	162	30	117	119	33	124	112	29	82	146	50	85	134	49	123	107	22
22	66	14	1	50	7	3	76	27	7	46	9	6	65	16	5	78	22	9	68	41	3
23	19	5	0	15	4	0	16	4	1	18	5	0	23	8	0	32	11	2	13	4	0
31	19	49	12	24	26	2	34	32	8	34	43	10	26	62	16	32	42	15	34	23	8
32	17	2	5	17	4	0	6	8	0	18	1	0	21	6	1	22	19	2	12	4	0
33	2	0	0	0	0	0	1	0	0	3	0	1	2	0	0	2	2	0	0	0	0
f_t	535	417	108	606	404	73	575	405	89	626	353	110	548	399	134	528	422	136	601	385	82
f_t^P	538	423	108	610	406	73	582	408	80	630	357	110	552	403	134	529	428	136	606	389	84

	133			134			135			136			137			137A			138		
1	204	310	47	243	317	58	237	326	27	271	266	57	313	256	54	67	319	58	198	333	34
2	298	61	27	325	67	20	326	78	10	262	57	23	252	36	16	311	158	47	331	106	13
3	67	16	2	54	24	1	28	9	0	59	21	4	60	10	0	65	40	2	37	10	0
11	69	111	23	83	137	21	83	142	11	125	122	23	146	135	20	10	49	8	67	114	15
12	235	51	21	256	49	13	256	58	9	198	58	9	215	27	13	186	100	31	252	71	8
13	45	10	2	38	19	1	20	7	0	42	13	2	45	9	0	34	23	1	27	7	0
21	109	153	30	132	157	30	140	169	15	123	109	37	125	90	26	43	221	43	118	193	19
22	48	6	5	49	15	3	59	17	1	43	11	2	28	5	3	105	42	11	69	32	5
23	20	6	0	15	5	0	8	2	0	17	4	2	15	0	0	30	16	1	10	3	0
31	24	39	4	25	22	6	12	15	1	20	30	7	27	26	5	13	45	7	11	25	0
32	12	4	0	17	3	4	7	2	0	18	1	1	7	2	0	20	15	5	8	2	0
33	2	0	0	1	0	0	0	0	0	0	4	0	0	0	0	1	1	0	0	0	0
f_t	571	385	85	620	412	79	590	414	37	594	342	84	623	304	70	444	516	107	565	450	47
f_t^P	578	390	85	629	413	79	594	418	37	600	347	84	648	306	71	448	518	107	569	452	47

Appendix 3-1 (cont.)

	139			140A			141			142			143			144		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	270	310	32	197	341	35	219	278	73	213	337	42	111	319	73	92	329	92
2	306	103	5	329	69	24	285	60	17	340	67	14	324	111	23	339	64	25
3	30	7	0	44	16	1	65	25	1	39	17	0	66	30	0	84	33	3
11	115	142	13	57	129	10	80	115	22	76	116	19	26	77	8	15	65	11
12	212	88	3	257	59	18	217	47	12	275	50	10	222	76	17	262	48	18
13	25	7	0	27	8	0	52	20	1	29	13	0	51	22	0	61	28	3
21	143	147	15	123	182	23	108	134	38	124	194	19	72	202	49	63	201	68
22	86	15	2	55	10	4	48	8	4	48	16	3	79	26	4	51	9	4
23	5	0	0	15	8	1	12	5	0	10	4	0	15	8	0	21	4	0
31	10	15	4	16	26	2	29	26	10	11	25	3	13	37	15	14	57	13
32	7	0	0	14	0	2	19	5	1	16	1	0	20	7	2	23	7	3
33	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	2	1	0
f	612	414	37	573	422	61	570	362	91	592	421	56	503	458	96	513	428	120
f_t^p	614	421	37	575	429	61	577	364	91	596	423	56	505	465	96	521	429	120

Appendix 3-2: Matrices of transition frequencies (in numbers of grains) for the sequences of grains which are similar to chains of the first order and have a matrix of transition probabilities different from that of ideal granite. A-stock; B-dykes.

A

	3			17			36			44			45			52			96		
1	130	324	92	147	224	109	128	287	145	182	195	69	172	252	100	251	286	71	303	258	87
2	334	82	20	235	131	34	299	50	32	308	138	13	272	77	37	296	86	20	239	32	40
3	79	32	2	94	49	7	128	48	4	58	24	0	82	55	6	61	32	1	102	27	4
11	34	69	27	50	67	30	34	60	33	60	96	25	58	85	27	94	121	35	139	118	44
12	244	59	14	129	72	21	219	39	25	191	93	9	178	50	23	208	63	11	194	24	34
13	66	23	2	67	35	6	100	43	1	51	18	0	58	38	4	44	27	0	63	22	2
21	73	200	56	72	105	51	57	156	80	98	167	39	83	134	52	130	136	27	114	88	33
22	61	17	3	76	45	9	39	5	4	99	35	3	52	15	8	62	18	6	23	6	3
23	13	7	0	22	11	0	25	5	2	7	6	0	23	13	1	14	5	1	33	4	2
31	22	48	9	24	46	21	35	62	29	24	29	4	29	33	20	25	27	9	44	48	9
32	24	6	2	28	14	4	39	6	3	17	7	0	37	12	6	24	5	3	19	1	3
33	0	2	0	5	2	0	3	0	1	0	0	0	1	4	1	1	0	0	3	1	0
f	546	436	113	480	400	150	560	381	180	546	459	82	524	386	143	608	402	94	648	311	133
f_t^p	551	444	114	490	406	152	569	387	181	552	460	82	529	389	143	613	406	94	655	321	134

	112
1	116	339	68
2	322	111	49
3	84	34	4
11	22	78	16
12	227	73	34
13	46	20	2
21	73	205	41
22	69	28	14
23	33	14	2
31	21	53	10
32	24	8	1
33	4	0	0
f	523	482	122
f_t^p	526	486	122

B

	31			140			147		
	1	2	3	1	2	3	1	2	3
1	55	225	175	51	261	62	194	366	23
2	242	58	70	244	160	102	357	79	16
3	150	91	21	79	85	5	33	6	1
11	9	27	19	12	30	9	64	121	8
12	134	31	35	134	85	40	286	64	11
13	96	57	15	32	27	3	19	4	0
21	29	122	90	30	170	40	114	223	15
22	37	10	11	72	49	38	64	13	2
23	44	22	4	44	55	2	13	2	1
31	17	69	63	7	60	12	16	17	0
32	49	17	24	37	25	22	5	0	1
33	10	9	2	2	3	0	1	0	0
f	435	370	262	374	506	169	583	432	40
f_t^p	456	376	269	378	510	170	589	457	49

Appendix 3-3: Matrices of transition frequencies (in numbers of grains) for the sequences of grains which are similar to chains of the second order. A-stock; B-dykes.

A

	1			10			11			25			28			29			30		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	225	309	98	126	208	126	91	252	120	87	245	145	21	152	153	92	240	109	50	186	155
2	314	19	14	223	73	77	275	70	52	250	39	59	152	144	146	235	116	92	184	144	104
3	94	18	0	110	94	37	102	71	13	141	68	12	155	145	73	113	87	14	158	100	38
11	94	88	40	37	52	36	15	47	28	24	43	20	1	9	10	25	53	14	13	15	22
12	279	18	9	120	50	35	191	35	25	177	28	35	69	50	32	139	62	37	92	50	43
13	82	16	0	61	48	17	76	38	6	100	38	5	65	61	25	60	41	7	87	51	15
21	100	169	35	64	101	57	59	154	58	45	125	78	9	71	70	46	135	52	20	89	71
22	15	1	3	38	15	19		15	14	23	8	8	36	49	59	55	29	31	49	59	34
23	12	2	0	35	33	9	24	24	4	31	23	5	60	51	35	46	39	7	52	38	14
31	29	47	17	23	52	33	15	49	34	17	74	44	11	71	73	20	48	43	17	77	61
32	16	0	1	63	8	22	40	18	73	48	3	16	45	45	55	37	23	24	41	34	25
33	0	0	0	13	13	11	2	9	2	7	3	2	30	30	13	7	7	0	19	11	8
111	43	33	15	9	14	13	3	8	4	4	13	7	0	1	0	6	19	0	2	4	7
112	77	8	1	26	20	6	32	10	5	30	6	7	6	2	1	29	14	9	4	5	6
113	31	9	0	18	12	6	17	11	0	14	6	0	2	5	2	7	6	1	12	8	1
121	83	156	30	30	59	30	37	107	43	32	84	60	7	37	25	25	83	31	9	43	37
122	14	1	3	28	8	13	22	5	8	18	5	5	15	16	19	30	16	16	20	19	10
123	8	1	0	19	12	4	14	10	1	19	14	2	11	13	8	21	13	3	23	17	3
131	25	43	13	11	30	20	10	34	31	11	49	36	5	31	29	11	25	23	9	43	32
132	14	0	1	29	4	14	23	9	6	31	1	5	20	15	26	15	12	13	18	21	12
133	0	0	0	5	7	5	1	4	1	4	1	0	12	8	5	4	3	0	8	3	4
211	41	41	18	20	26	18	10	26	22	15	20	10	0	2	5	14	25	7	4	7	9
212	155	7	7	58	20	21	119	19	16	93	16	13	38	19	13	82	34	18	51	18	20
213	31	4	0	28	23	6	34	19	5	52	21	3	33	29	8	26	21	5	36	26	8
221	9	4	2	12	18	13	10	23	6	5	11	7	0	15	21	10	31	14	7	23	18
222	1	0	0	8	3	4	6	5	3	3	2	3	11	15	23	14	7	7	16	23	19
223	3	0	0	8	9	2	4	8	2	7	1	0	24	19	16	14	15	2	18	9	7
231	4	4	4	8	15	10	5	13	3	4	18	7	4	25	31	7	21	17	7	25	20
232	2	0	0	24	3	6	14	6	4	14	2	7	16	20	15	18	9	10	19	10	9
233	0	0	0	4	3	2	1	3	0	3	1	1	13	15	7	3	4	0	7	6	1
311	9	14	6	8	10	5	2	11	2	3	1	1	1	6	4	4	9	7	7	4	6
312	42	2	1	33	10	8	38	6	4	4	10	3	25	28	18	24	14	10	35	25	16
313	16	1	0	15	13	5	25	8	1	52	5	15	30	27	15	27	14	1	38	17	6
321	7	7	2	22	28	13	10	22	8	7	29	11	2	17	24	10	18	7	4	22	15
322	0	0	0	2	4	2	9	5	3	2	1	0	10	18	17	11	4	8	12	17	5
323	1	0	0	8	12	2	6	6	1	5	8	3	25	19	11	11	11	2	11	11	3
331	0	0	0	4	7	2	0	2	0	1	5	1	2	15	13	2	2	3	1	9	9
332	0	0	0	10	1	2	3	3	3	1	0	2	7	10	13	4	2	1	4	3	4
333	0	0	0	4	3	4	0	1	1	0	1	1	5	7	1	0	0	0	3	2	3
f_t	632	347	112	460	373	241	463	397	186	477	348	221	326	442	373	441	443	214	391	432	296
f_t^p	646	351	112	464	378	241	472	400	186	485	354	223	329	443	375	445	449	215	398	435	298

	37			47			49			58			73			87			115		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	109	247	134	199	283	61	200	307	72	209	265	87	123	257	75	109	224	134	33	137	192
2	257	53	65	269	106	57	301	69	29	275	77	34	261	166	49	221	99	78	144	29	91
3	127	72	26	74	44	2	77	24	5	79	43	1	68	55	4	140	75	11	187	97	89
11	25	55	29	75	99	25	70	107	23	80	89	36	41	58	23	25	50	32	0	11	21
12	175	36	35	184	72	25	234	45	24	187	55	20	144	94	19	143	49	32	87	11	36
13	73	42	19	39	22	0	51	18	3	53	34	0	43	28	3	87	43	4	102	36	51
21	60	138	53	99	140	25	105	163	29	96	142	34	63	160	38	54	107	60	12	56	73
22	36	6	11	69	24	12	44	20	2	54	18	5	93	49	23	52	23	21	15	4	10
23	42	20	3	33	21	2	23	4	2	25	8	1	25	23	1	43	29	6	45	25	19
31	24	50	52	22	40	10	25	32	20	33	30	16	19	35	14	30	65	42	21	64	97
32	42	10	18	14	10	19	19	2	3	30	3	9	24	23	7	24	26	25	39	14	40
33	12	10	4	2	0	0	3	2	0	0	1	0	0	4	0	8	2	1	38	31	18
111	7	14	7	30	34	11	20	37	13	25	37	16	14	19	7	4	13	8	0	0	0
112	38	8	9	53	31	13	79	16	11	16	18	5	25	27	6	24	14	8	8	0	3
113	18	10	1	17	8	0	18	5	1	25	11	0	14	7	1	21	10	1	14	3	4
121	23	4	9	69	97	14	85	125	21	65	94	25	31	91	22	31	73	39	11	34	40
122	23	4	9	45	16	10	25	15	2	38	13	4	50	31	12	24	11	12	5	2	4
123	20	12	3	16	9	0	19	3	2	16	3	1	9	10	9	15	15	2	18	10	8
131	12	30	30	10	26	3	15	19	17	24	22	7	13	24	6	16	42	27	10	36	51
132	25	6	11	7	6	8	15	1	2	24	3	6	10	12	6	10	14	19	15	7	14
133	9	7	3	0	0	0	2	1	0	0	0	0	0	3	0	2	1	1	20	16	13
211	16	27	17	36	54	9	40	58	7	41	39	15	21	28	14	13	26	15	0	3	8
212	98	21	19	99	33	8	125	24	13	98	28	13	101	50	9	75	19	13	29	6	18
213	33	13	7	15	10	0	19	8	2	19	15	0	23	14	1	36	22	2	39	13	18
221	16	16	3	24	36	8	16	22	6	19	31	4	24	55	14	16	25	11	0	5	10
222	6	0	0	19	5	0	16	4	0	14	4	0	31	11	7	15	5	3	2	0	2
223	7	4	0	9	2	1	2	0	0	3	2	0	12	10	1	14	6	1	5	5	0
231	9	17	16	12	13	6	9	11	3	9	8	8	6	11	8	12	17	13	5	15	25
232	11	3	5	6	4	11	3	1	0	5	0	3	12	9	1	13	11	5	12	3	10
233	2	0	1	2	0	0	1	1	0	0	1	0	0	1	0	6	0	0	10	7	2
311	5	14	5	8	10	4	10	12	3	14	13	5	6	11	2	8	11	9	0	8	13
312	37	5	7	29	7	4	26	4	0	22	6	2	16	16	3	39	15	11	47	4	13
313	22	19	11	6	4	0	15	5	0	8	8	0	6	7	1	30	11	1	49	20	28
321	8	26	8	5	6	3	4	12	2	11	14	5	8	14	2	7	8	9	1	15	22
322	7	1	2	5	3	2	7	1	0	1	1	1	12	7	4	13	6	6	8	2	4
323	14	4	0	8	10	1	2	1	0	6	3	0	4	3	0	14	8	3	18	9	11
331	3	3	6	0	1	1	1	2	0	0	0	0	0	0	0	2	4	2	4	13	21
332	7	1	2	0	0	0	1	0	1	1	0	0	2	2	0	1	1	0	11	4	15
333	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	7	8	3
f_t	490	375	225	543	431	120	579	399	106	561	386	123	455	476	127	467	398	226	362	264	370
f_t^p	497	377	225	550	435	120	583	406	106	568	390	123	456	478	128	472	401	226	371	268	377

Appendix 3-3 (cont.)

B	121			123			132			141A			145			146		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1	143	268	120	206	286	80	140	245	110	152	347	59	168	277	90	212	322	43
2	266	79	42	282	64	38	250	65	70	345	68	20	287	76	51	323	120	19
3	121	42	9	86	36	10	102	86	16	65	77	2	82	57	6	42	20	0
11	35	73	33	78	95	32	38	75	25	43	93	15	51	84	31	66	131	13
12	189	50	25	214	46	21	164	37	41	276	57	13	191	55	30	233	77	8
13	86	28	6	57	18	6	61	42	6	48	10	1	60	28	1	33	10	0
21	72	140	51	101	141	37	69	114	67	92	206	40	88	155	40	127	170	23
22	55	20	4	49	7	8	32	8	22	53	11	4	55	9	10	78	33	8
23	29	12	1	23	12	3	32	39	8	15	4	1	22	24	4	9	10	0
31	35	51	35	26	48	11	31	53	17	16	44	3	27	35	18	17	18	6
32	20	9	12	17	11	8	52	19	15	14	0	3	37	9	11	10	7	3
33	5	2	2	5	5	0	9	5	2	0	2	0	0	5	1	0	0	0
111	7	18	8	30	37	10	9	23	6	12	29	2	16	22	12	22	39	5
112	45	16	11	74	16	4	46	14	13	72	14	7	54	18	11	95	30	5
113	26	5	2	23	6	3	11	12	1	12	3	0	19	11	0	11	2	0
121	44	106	37	72	112	28	45	71	48	69	172	29	61	101	27	85	128	17
122	35	13	2	35	6	5	18	6	11	43	10	4	41	7	5	54	18	5
123	18	7	0	11	8	2	15	26	0	11	2	0	17	11	1	6	2	0
131	21	41	24	18	30	7	18	31	12	11	32	3	22	24	12	12	15	5
132	15	4	9	10	4	4	20	13	9	8	0	2	17	6	5	5	3	2
133	4	0	2	3	3	0	4	1	1	0	1	0	0	0	1	0	0	0
211	21	35	16	35	51	15	20	33	15	23	57	11	26	45	16	37	83	5
213	105	27	7	105	23	9	77	15	21	165	36	4	110	30	15	124	42	3
221	34	13	4	25	9	3	42	22	3	32	7	1	30	10	0	17	6	0
222	22	20	12	21	20	7	7	19	6	17	26	9	15	33	5	38	34	6
223	14	5	1	5	1	1	4	1	3	10	1	0	7	0	2	18	12	2
231	3	1	0	6	2	0	12	7	3	2	1	1	3	7	9	3	5	0
232	12	7	10	7	12	4	10	16	5	4	11	0	5	11	6	5	3	1
233	5	4	2	5	3	4	28	5	6	3	0	1	16	3	5	5	4	1
311	0	1	0	1	2	0	3	4	1	0	1	0	0	4	0	0	0	0
312	6	20	9	13	6	7	8	18	4	8	7	1	9	15	3	7	7	3
313	37	6	6	33	7	8	40	7	6	35	7	2	25	6	4	13	3	0
321	25	10	0	8	3	0	7	8	2	3	0	0	10	7	1	5	1	0
322	6	12	2	7	8	2	7	8	2	10	1	7	0	0	0	0	2	1
323	6	2	1	9	0	2	5	5	5	2	1	0	2	6	3	0	3	0
331	7	4	1	5	2	1	3	6	0	0	0	0	0	0	0	0	0	0
332	2	2	1	0	5	0	4	1	0	2	0	0	4	0	1	0	0	0
333	0	1	1	2	3	0	2	0	0	0	0	0	0	1	0	0	0	0
f_t	531	387	172	572	384	132	495	394	204	558	433	84	535	414	145	577	462	62
f_t^P	536	392	172	477	389	132	498	400	205	568	434	84	543	417	147	583	467	62

APPENDIX 4

Values of c_{ij} of the elements of the matrix C. A-stock; B-dyke.

Specimen	C_{OrOr}	C_{QQ}	C_{AbAb}	$C_{OrQ}=C_{QOr}$	$C_{OrAb}=C_{AbOr}$	$C_{AbQ}=C_{QAb}$
A						
2	0.6624	0.4785	0.1899	1.4044	1.3017	0.7803
4	0.6018	0.4641	0.2763	1.4600	1.4991	0.4444
5	0.3368	0.6724	0.3791	1.3959	1.4388	0.9118
6	0.4752	0.5530	0.2944	1.3752	1.3268	1.0657
7	0.3150	0.6299	0.2700	1.4346	1.5668	0.7973
8	0.7484	0.3690	0.1550	1.3800	1.2395	0.7894
9	0.6195	0.7297	0.1995	1.2970	1.4855	0.7046
12	0.5603	0.6144	0.2825	1.3289	1.3474	0.9887
13	0.6402	0.5232	0.2202	1.3858	1.2907	0.8737
14	0.4854	0.3187	0.0442	1.5575	1.5112	0.6324
15	0.7266	0.4788	0.2054	1.3283	1.2686	0.9372
19	0.7332	0.1603	0.0000	1.4589	1.2649	0.7372
20	0.3540	0.8635	0.7117	1.1853	1.3147	1.0158
21	0.7209	0.4381	0.1757	1.3538	1.2903	0.8756
22	0.8456	0.3397	0.3102	1.3082	1.0898	0.9905
23	0.6990	0.2717	0.0000	1.4567	1.4204	0.4828
24	0.5560	0.4474	0.1311	1.4500	1.3834	0.8193
26	0.7124	0.7519	0.5220	1.2360	1.1873	0.9856
27	0.7216	0.6999	0.2897	1.2692	1.2237	0.9290
32	0.6825	0.4946	0.2662	1.3957	1.3952	0.5616
33	0.5405	0.5446	0.5146	1.3337	1.2212	1.1119
34	0.5672	0.3855	0.0931	1.4970	1.3149	0.8105
35	0.4574	0.5609	0.3188	1.3483	1.3106	1.1300
38	0.6465	0.3510	0.2008	1.4705	1.4195	0.5245
39	0.6114	0.4266	0.0561	1.4427	1.4132	0.7159
40	0.3478	0.6033	0.3241	1.5036	1.5536	0.6125
41	0.6982	0.3120	0.0000	1.4485	1.3468	0.5986
42	0.7898	0.1964	0.0000	1.3908	1.2806	0.6981
43	0.5778	0.4803	0.4486	1.4035	1.2222	1.0290
46	0.6281	0.5404	0.3299	1.4005	1.1954	0.9160
48	0.7826	0.7135	0.4723	1.2485	1.3182	0.6486
50	0.7511	0.6998	0.4476	1.2687	1.1427	0.9154
51	0.7092	0.6043	0.5656	1.3167	1.0767	1.0612
53	0.6105	0.4717	0.2022	1.4249	1.3572	0.7848
54	0.4997	0.5224	0.1415	1.4876	1.6454	0.3787
55	0.6014	0.4783	0.2018	1.4358	1.4548	0.6153
56	0.7445	0.4225	0.5555	1.3734	1.1215	0.9336
57	0.7569	0.5346	0.3083	1.3246	1.2564	0.7739
59	0.5611	0.4186	0.2461	1.4639	1.3224	0.8762
60	0.5510	0.6731	0.3534	1.3115	1.3383	0.9736
61	0.5568	0.3714	0.0000	1.5024	1.3644	0.7903
62	0.6500	0.6630	0.0604	1.3137	1.3612	0.8542
65	0.7214	0.5265	0.0000	1.3424	1.2878	0.8391
66	0.4657	0.4999	0.2125	1.5067	1.4548	0.6570
67	0.5954	0.2949	0.1390	1.5287	1.4914	0.4190
68	0.5433	0.5075	0.2412	1.4738	1.5841	0.3973
70	0.6690	0.2826	0.1403	1.4829	1.4680	0.3674
71	0.6765	0.6585	0.1586	1.3220	1.4392	0.6345
72	0.6167	0.7628	0.0000	1.3060	1.6442	0.4615
75	0.6877	0.6735	0.2821	1.3282	1.5426	0.3658
75	0.4277	0.5886	0.2553	1.4536	1.5363	0.6871
76	0.7517	0.7324	0.2222	1.2562	1.3991	0.6094
77	0.5968	0.5670	0.0000	1.4162	1.5664	0.4411
78	0.5822	0.2840	0.0000	1.5353	1.5471	0.3912
79	0.3591	0.4934	0.3330	1.5689	1.6410	0.4377
80	0.6836	0.7313	0.0000	1.2929	1.5916	0.4359
81	0.4481	0.5098	0.0000	1.5218	1.8111	0.2081
82	0.7449	0.5833	0.1662	1.3197	1.3477	0.6398
84	0.4582	0.5985	0.0000	1.4634	1.4583	0.6636

Appendix 4 (cont.)

Specimen	C_{OrOr}	C_{QQ}	C_{AbAb}	$C_{OrQ}=C_{QOr}$	$C_{OrAb}=C_{AbOr}$	$C_{AbQ}=C_{QAb}$
A						
85	0.3235	0.4545	0.1017	1.5913	1.4545	0.7333
86	0.6818	0.9944	0.2467	1.0067	1.4857	0.9978
88	0.6830	0.6785	0.0000	1.3211	1.6003	0.3692
89	0.7023	0.7864	0.6026	1.1963	1.2391	0.9737
90	0.4682	0.9650	0.3205	1.2032	1.6680	0.6622
91	0.7088	0.4364	0.2478	1.3642	1.1806	1.0369
92	0.7025	0.2997	0.0000	1.4434	1.3190	0.6851
93	0.8979	0.5232	0.3154	1.1798	1.0743	1.1866
94	0.5105	0.5082	0.3007	1.4158	1.2156	1.1062
95	0.6567	0.5602	0.3950	1.3770	1.2460	0.8353
97	0.6892	0.3561	0.2530	1.4440	1.4359	0.4135
98	0.6499	0.3472	0.0000	1.4562	1.3553	0.7302
99	0.5378	0.8353	0.3411	1.2388	1.4950	0.8172
100	0.6088	0.3924	0.0695	1.4608	1.3295	0.8178
101	0.6479	0.5443	0.4073	1.3666	1.2833	0.8693
102	0.4948	0.5889	0.4427	1.4001	1.3294	0.9187
103	0.6084	0.5634	0.4672	1.4111	1.4310	0.5647
104	0.7274	0.2908	0.0000	1.4320	1.2704	0.7053
105	0.5251	0.7832	0.2448	1.2701	1.4553	0.8829
106	0.5862	0.4790	0.2681	1.4411	1.3746	0.7419
107	0.5629	0.6046	0.3105	1.3824	1.2901	0.9201
108	0.5780	0.6817	0.4446	1.3686	1.5096	0.5898
110	0.3651	0.8177	0.2087	1.3562	1.7397	0.5498
111	0.9045	0.6584	0.1429	1.1675	1.1515	0.9054
113	0.7247	0.5606	0.0000	1.3388	1.4581	0.5091
114	0.5881	0.8795	0.6617	1.2675	1.3931	0.7003
116	0.2364	1.7600	0.7962	1.0926	1.4704	0.6854
117	0.3972	1.2456	0.8119	1.2142	1.4789	0.5809
B						
63	0.7442	0.5062	0.1943	1.3483	1.2641	0.7425
64	0.6224	0.6399	0.0000	1.3680	1.5998	0.427
109	0.4644	0.3220	0.0000	1.5954	1.4423	0.6093
118	0.7178	0.5563	0.3151	1.3284	1.4172	0.5794
119	0.7218	0.5966	9.3547	1.3209	1.3696	0.6455
120	0.5910	0.6151	0.1915	1.3728	1.4391	0.7232
122	0.7759	0.7484	0.5666	1.2311	1.2570	0.7778
124	0.6269	0.5137	0.1554	1.4184	1.5020	0.4907
125	0.5241	0.4996	0.1808	1.4788	1.5084	0.5598
126	0.6764	0.3987	0.0000	1.4323	1.2942	0.7393
127	0.7386	0.7325	0.1378	1.2651	1.4826	0.5043
128	0.7535	0.5454	0.3644	1.3294	1.3460	0.5844
129	0.5759	0.5753	0.1198	1.4097	1.5144	0.5891
130	0.6599	0.6583	0.2344	1.3175	1.3351	0.8274
131	0.8277	0.8105	0.0000	1.1807	1.4147	0.5656
133	0.6523	0.4279	0.2987	1.4392	1.3463	0.6412
134	0.6982	0.4423	0.1794	1.4023	1.2700	0.7510
135	0.7067	0.4756	0.0000	1.3901	1.3110	0.6461
137	0.7890	0.3941	0.0000	1.3418	1.3129	0.5890
137A	0.3640	0.6314	0.1878	1.4675	1.3847	0.8374
138	0.6563	0.5584	0.0000	1.3873	1.4237	0.5725
139	0.7766	0.6268	0.0000	1.2887	1.4653	0.4015
140A	0.6385	0.4042	0.3005	1.4594	1.2172	0.8061
141	0.6895	0.4686	0.1264	1.3957	1.3710	0.6355
142	0.6494	0.4044	0.0000	1.4520	1.3086	0.6991
143	0.4663	0.5563	0.0000	1.4751	1.5299	0.6276
144	0.3661	0.3760	0.2250	1.6148	1.5173	0.5972